Formalizing Hybrid Systems with Event-B

Jean-Raymond Abrial, Wen Su, Huibiao Zhu

August 2012

Prologue

Using B Formal Method in Industry

- Fully automatic train systems:
    - Paris metro line 14 (October 1998)
    - Roissy airport shuttle (March 2007)
- More train applications

Paris Metro Line 14 (Source: Siemens)

Line length	8.5 km
Number of Stops	8
Time interval between two trains	115 s
Speed	40 km/h
Number of trains	17
Passengers per day	350,000

Roissy Airport Shuttle (Source: Siemens)

Line length	3.3 km
Number of Stops	5
Time interval between two trains	105 s
Speed	26 km/h
Number of trains	14
Passengers per hour	$2,000$

Comparing the Case Studies (1)

	Paris	Roissy
Number of final ADA lines (from B)	$86,000$	$158,000$
Number of proofs	$27,800$	$43,610$
Percentage of interactive proofs	8.1	3.3
Interactive proofs in Man.Month	7.1	4.6

Comparing the Case Studies (2)

- Man.month calculated with:
    - 15 interactive proofs per man.day
    - 21 days in a month

- In both cases, no unit tests and no integration tests

- Reinforcing global tests (catastrophic scenarios)

-Important differences in the software requirements:
    - Paris: specially done for the project
    - Paris: adaptation from O'Hare Airport (problems)

Similar Applications by Siemens (1)

City	Line	Service	Driverless
Algiers	1	2011	No
Barcelona	9	2007	Yes
Budapest	4	2013	Yes
Caracas	4	2004	No
Helsinki	1	2013	No
Hong Kong	TKO	2001	No

Similar Applications by Siemens (2)

City	Line	Service	Driverless
Mexico	B	2000	No
New York	Canarsie	2006	No
	PATH	2014	No
Paris	14	1998	Yes
	3	2009	No
	1	2011	Yes
	5	2012	No

Similar Applications by Siemens (3)

City	Line	Service	Driverless
Rennes	B	2018	Yes
Roissy CDG	1	2007	Yes
	2	2007	Yes
San Juan	2	2004	No
Sao Paulo	TKO	2001	Yes

Contact: <Jean-Marc.Meynadier@siemens.com>

Similar Applications by Alstom (1)

System	City	Service	Size	Language	Driverless
KVB	French Trains	1993	30000	ADA	No
CDTC	Cairo	1996	3000	Modula2	No
SACEM	Paris (RER B)	1996	2500	Modula2	No
ACSES	AMTRACK (USA)	2002	14500	ADA	No

Similar Applications by Alstom (2)

System	City	Service	Size	Language	Driverless
Urbalis 200	Shanghai	2003	30000	ADA	No
	New Dehli				
	Seoul				
	Daegu				
	Incheoun				
	Madrid				
	Santiago				
	Cairo	2013			
	Bangalore				
	Calcutta				

Similar Applications by Alstom (3)

System	City	Service	Size	Language	Driverless
Urbalis 400	Shanghai	2008	100000	ADA	No
	Beijing				Yes
	Chenzen				No
	Sao Paulo	2013			Yes
	Mexico				No
	Milano				No
	Toronto				No
	Wuhan				No

Contact: <Luis-Fernando.Mejia@transport.alstom.com>

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- How can time be handled in Event-B?

Frequently Asked Questions about Event-B

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- Is it necessary to add a special "time feature" within Event-B?

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  where both discrete and continuous transitions can occur

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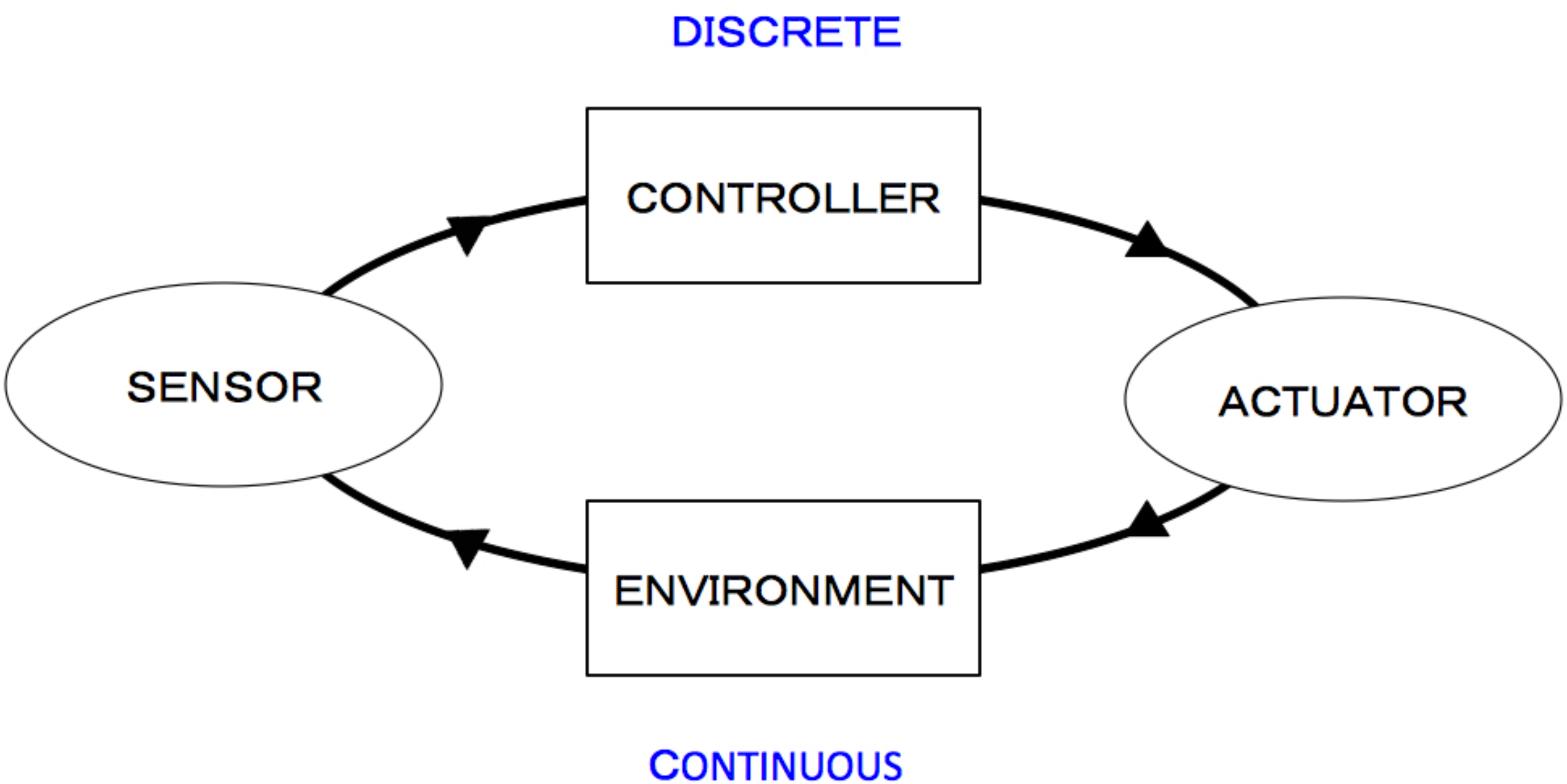
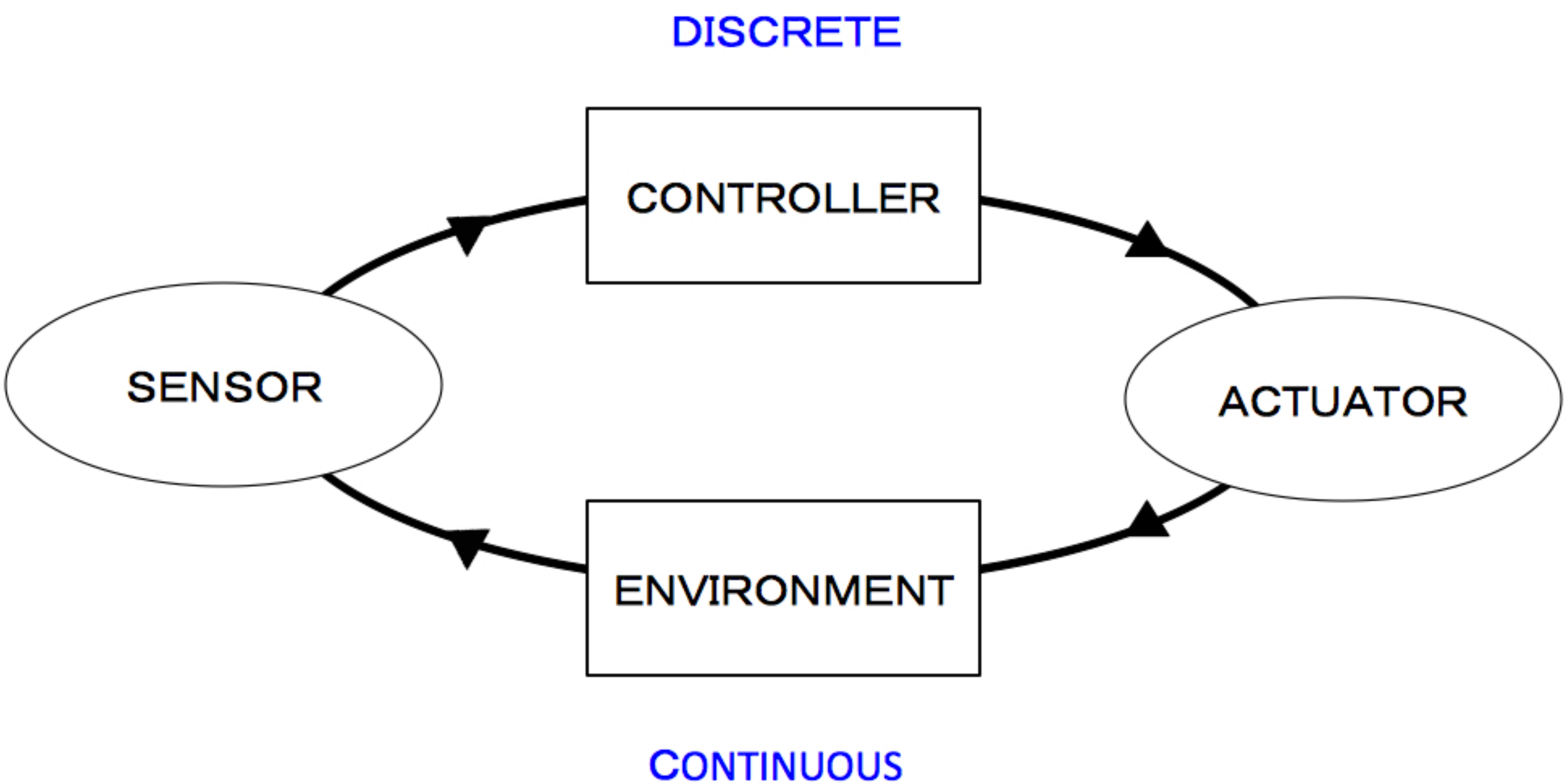
- Hybrid frameworks are frequent in embedded systems where:
  - A piece of software, the controller, manages an environment
  - Controller is linked to environment by sensors and actuators
  - Controller works from time to time in a DISCRETE fashion
  - While environment evolves in a CONTINUOUS way.

Illustration



Illustration



- We want to develop models of such closed systems

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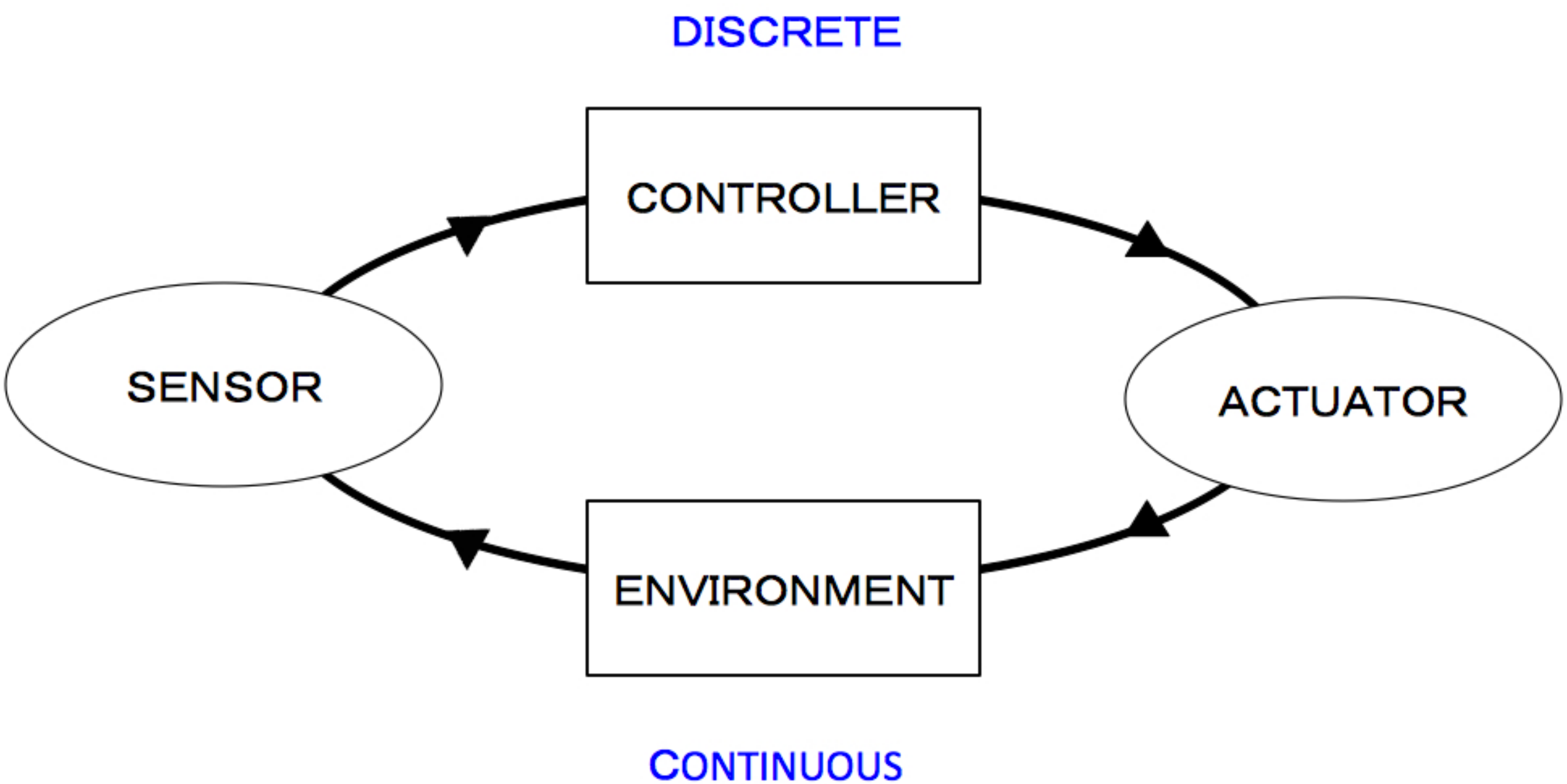

- We want to develop models of such closed systems

- We have thus to cope with both discrete and continuous evolutions

Example

- Continuous physical environment:

  a train defined by its position, speed, and acceleration

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- Discrete controller:

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- Goal: to control the speed of the train (station or another train)

Our Approach was influenced by that of Action System

R.J. Back and R. Kurki-Suonio.

Distributed Cooperation with Action Systems

ACM Transaction on Programming Languages and Systems. 1988.

R.J. Back, L. Petre, and I. Porres.

Generalizing Action Systems to Hybrid Systems.

FTRTFT 2000. LNCS 1926 Springer Verlag, 2000.

R.J. Back, C. Cerschi Seceleanu, and J. Westerholm.

Symbolic Simulation of Hybrid Systems.

APSEC'02, 2002.

Our Papers

Formalizing Hybrid Systems with Event-B

ABZ Conference. Pisa, June 2012

Complementary Methodologies for Developing Hybrid Systems with Event-B

Accepted at ICFEM 2012. Kyoto, November 2012

The Approach with Event-B (inspired by Action System)

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- We thus use refinement from a discrete to a continuous system

Three Examples

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- Examples developed and fully proved with the Rodin Platform
- These examples show complete analytical solutions

Example 1

- Two aircrafts are flying at the same altitude and speed

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- They might converge (collision) at some point o

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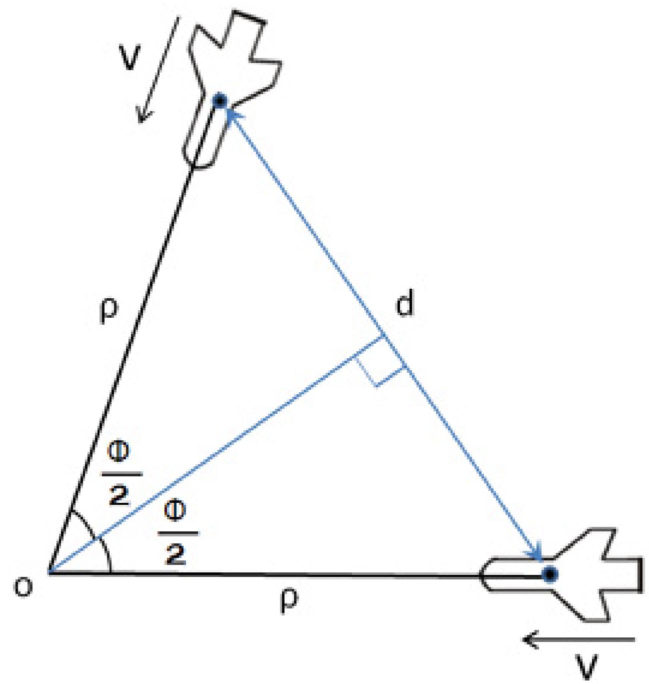

Aircraft Collision Avoidance: Constraint and goal

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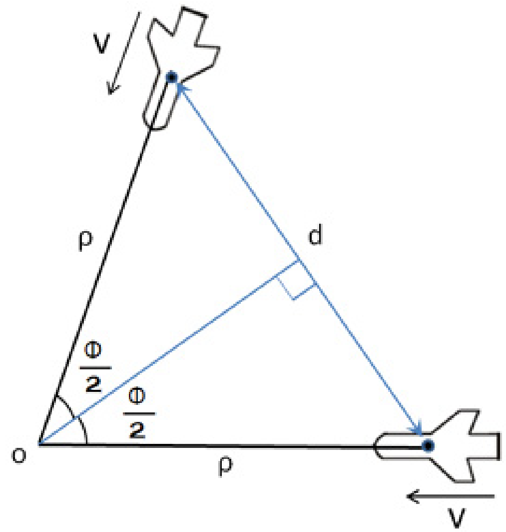

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$$d = 2\rho \sin \frac{\phi}{2}$$

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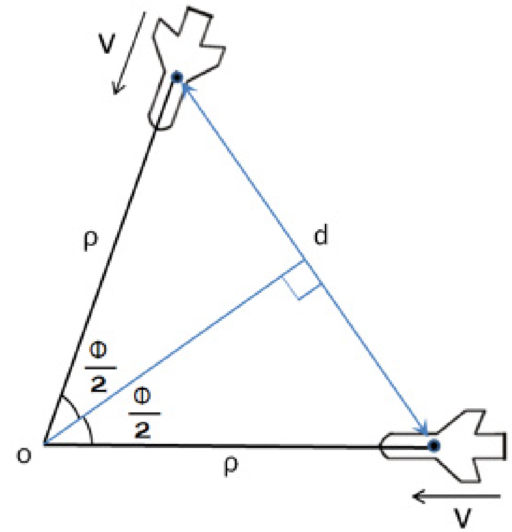

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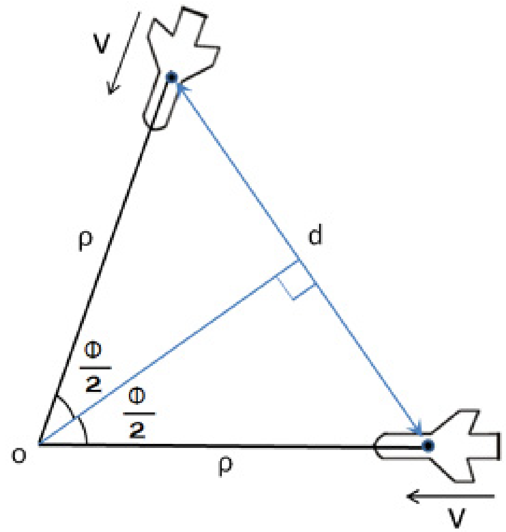

- The distance between aircrafts is as follows:

$$d = 2\rho \sin \frac{\phi}{2}$$

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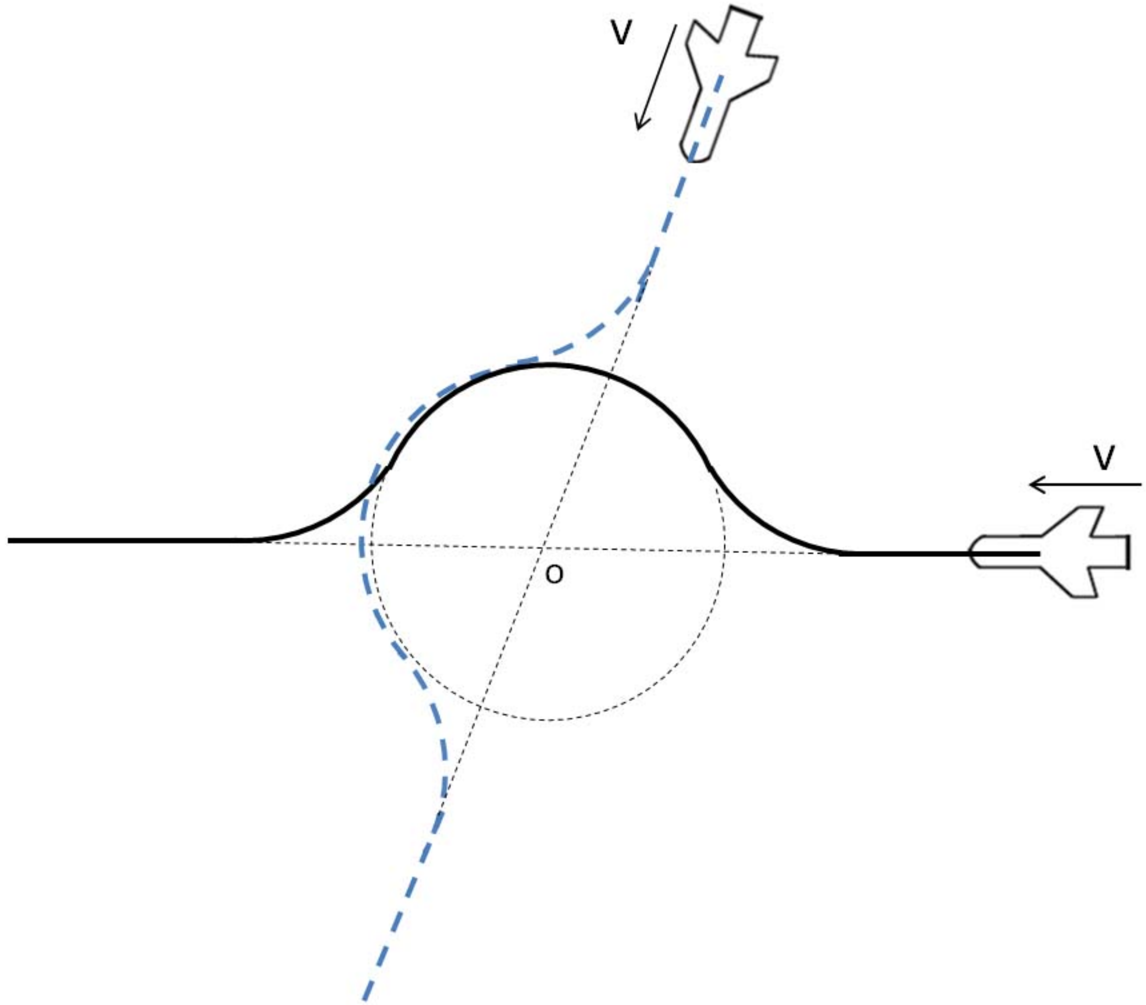
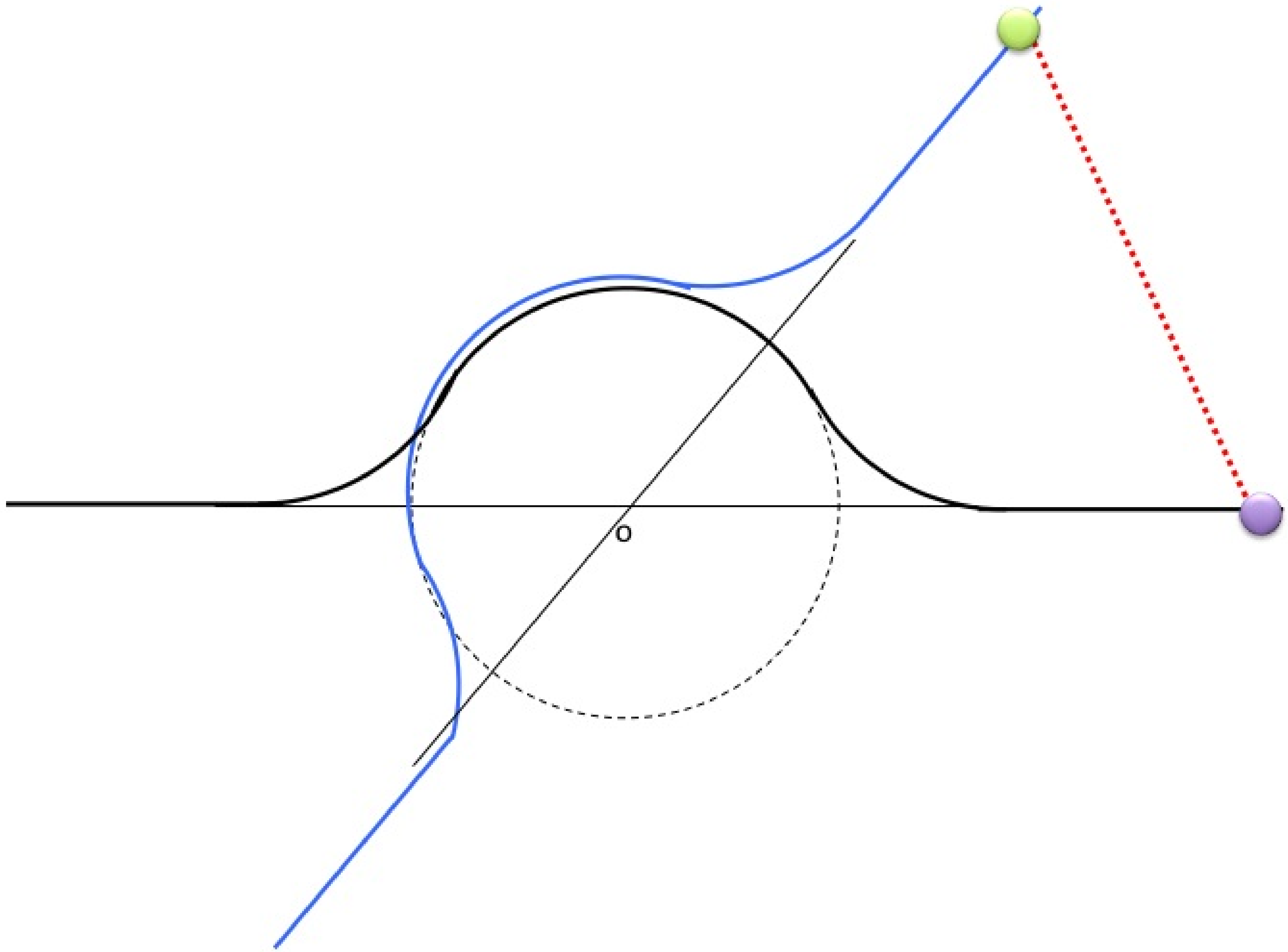
- Goal: we want to find a solution to avoid the collision

The Solution



- The radius r of this circle will be determined later

- Both aircrafts continue to fly at the same speed during the maneuver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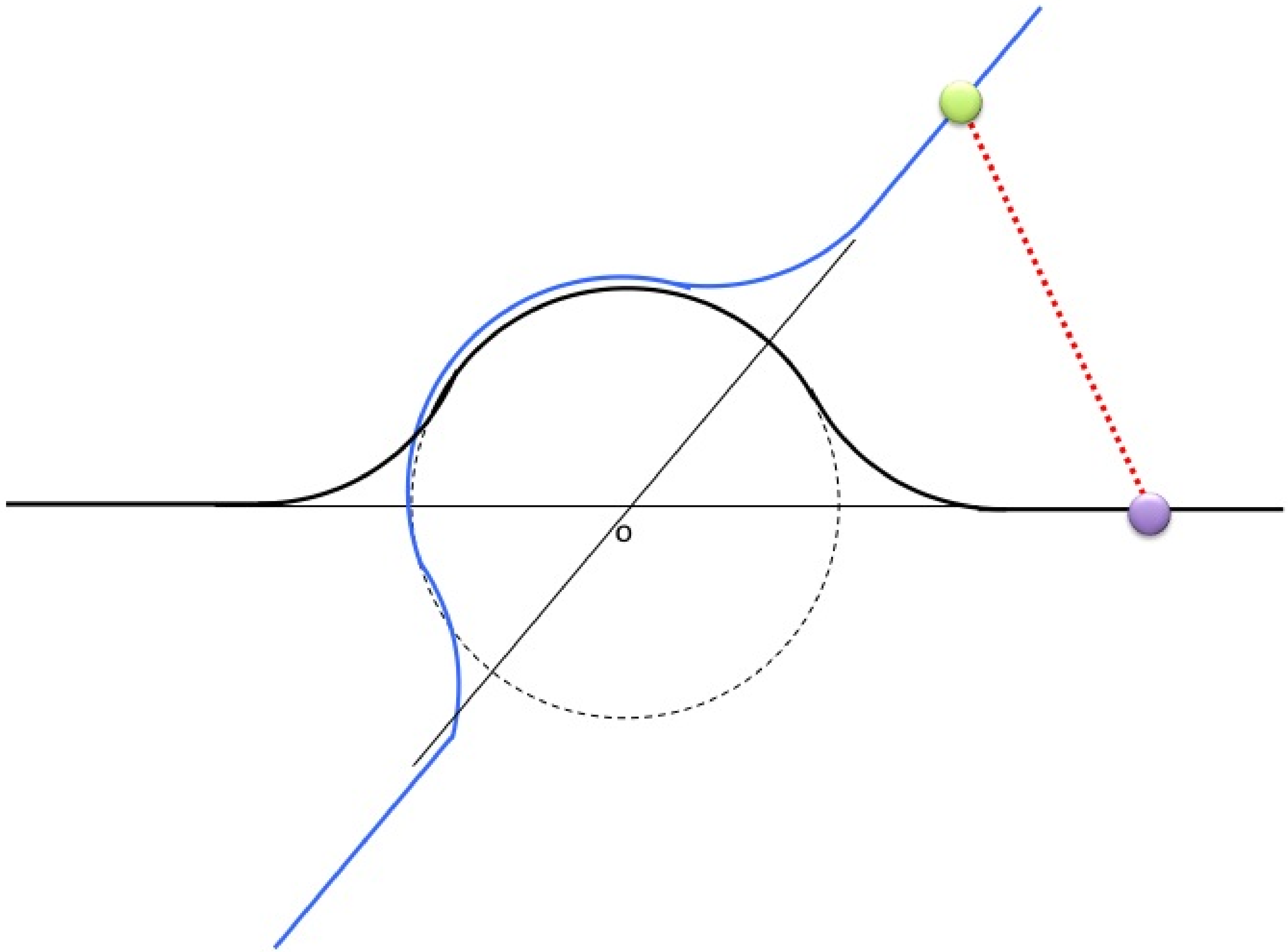
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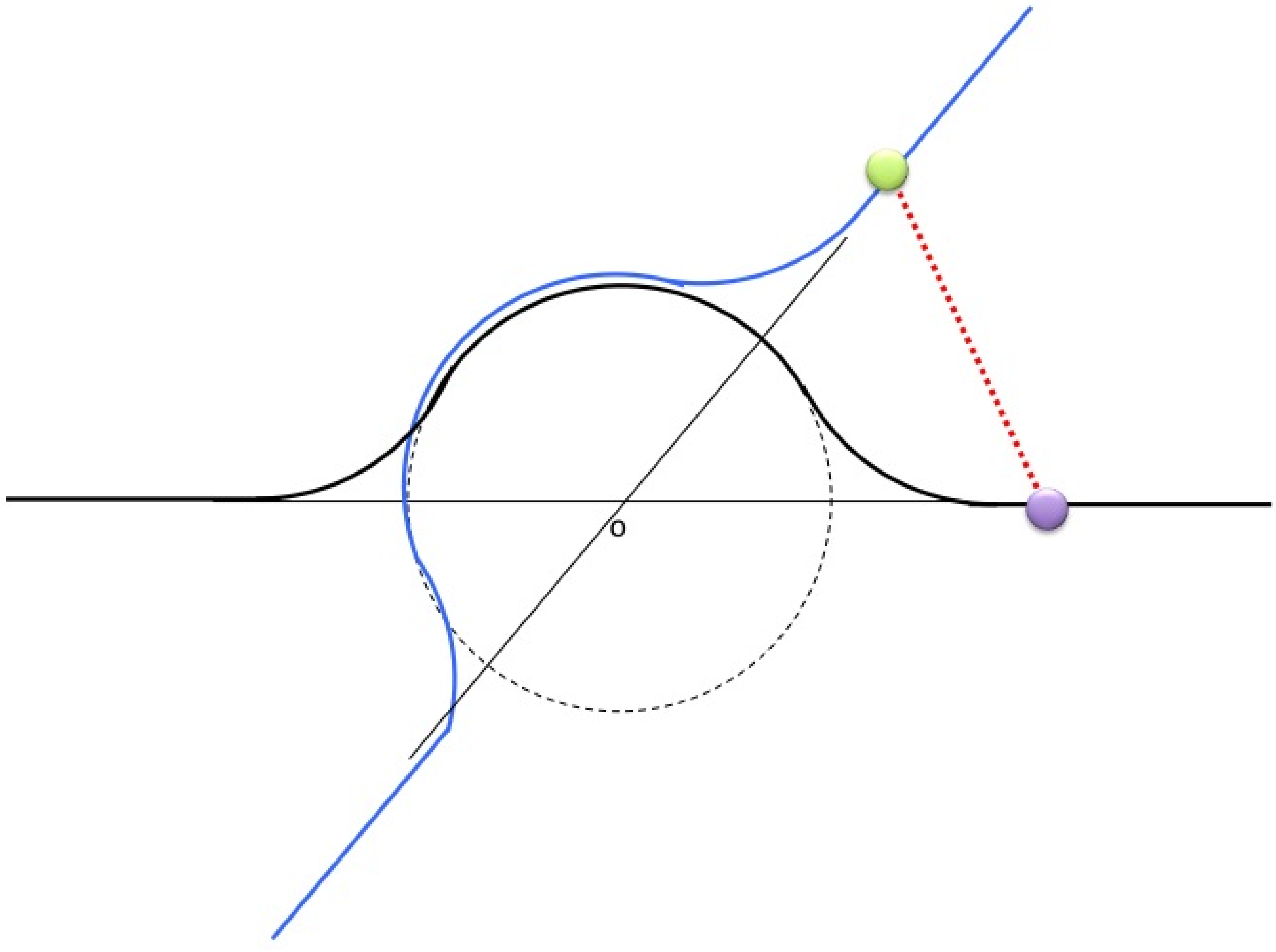
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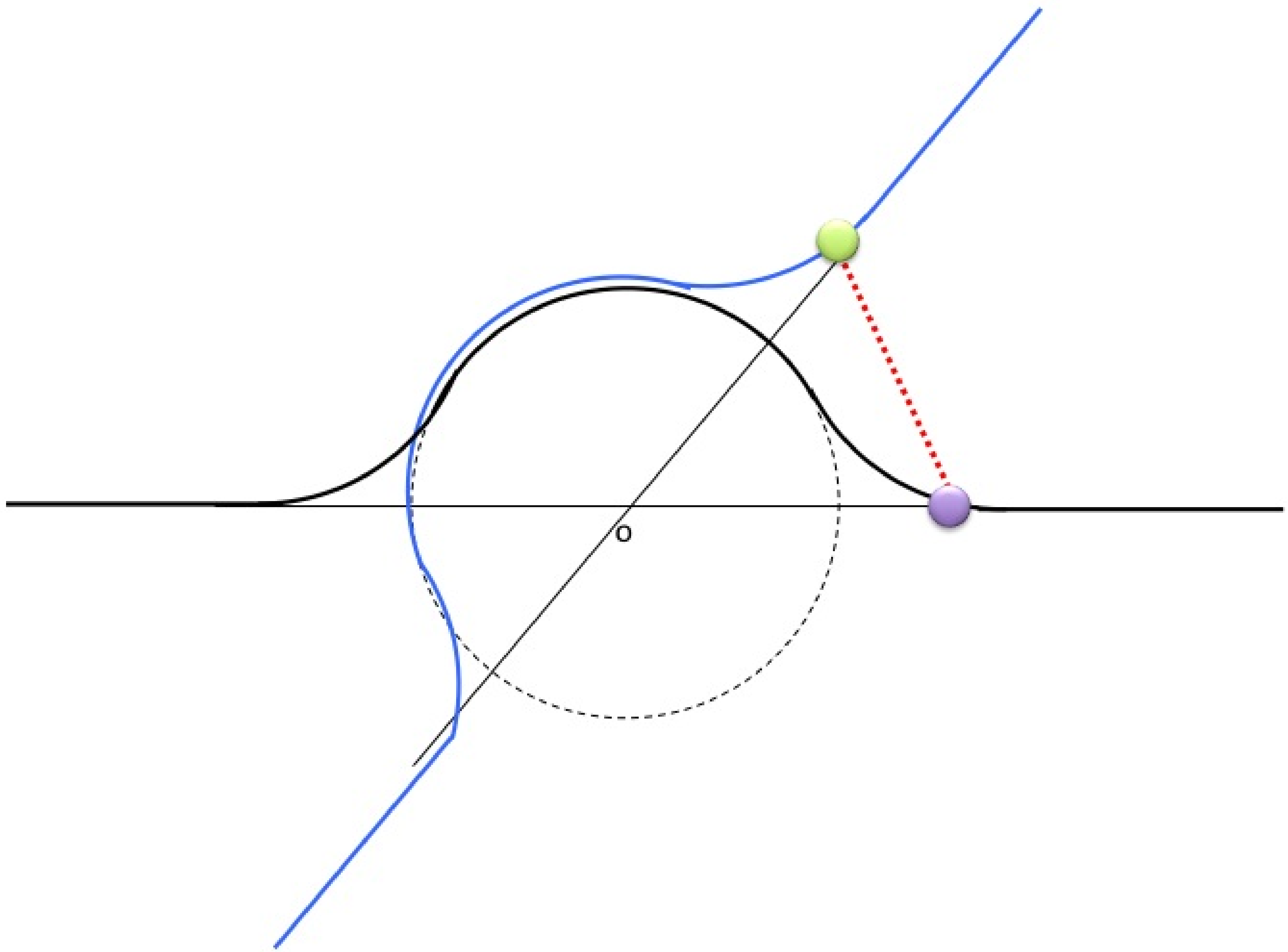
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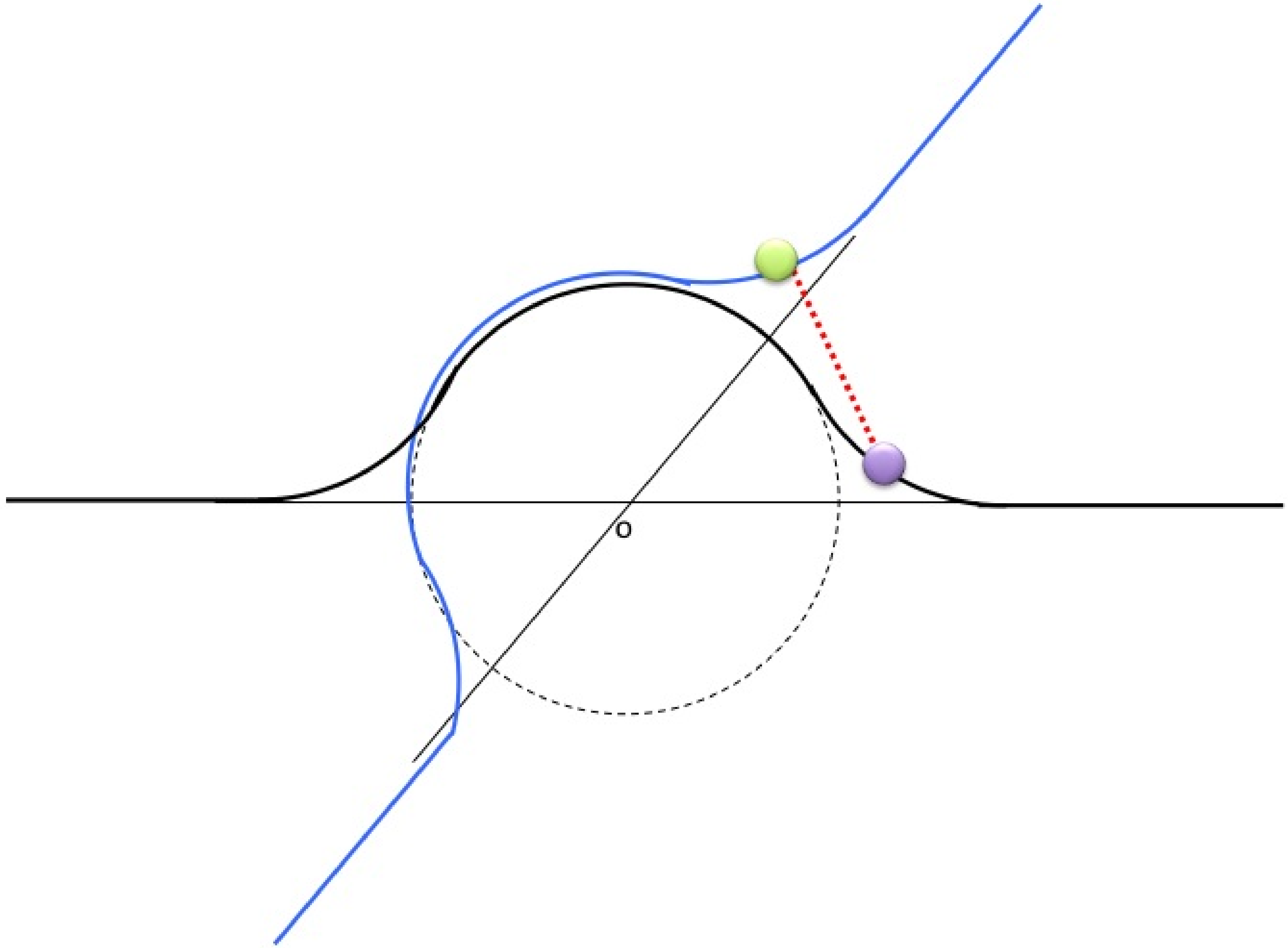
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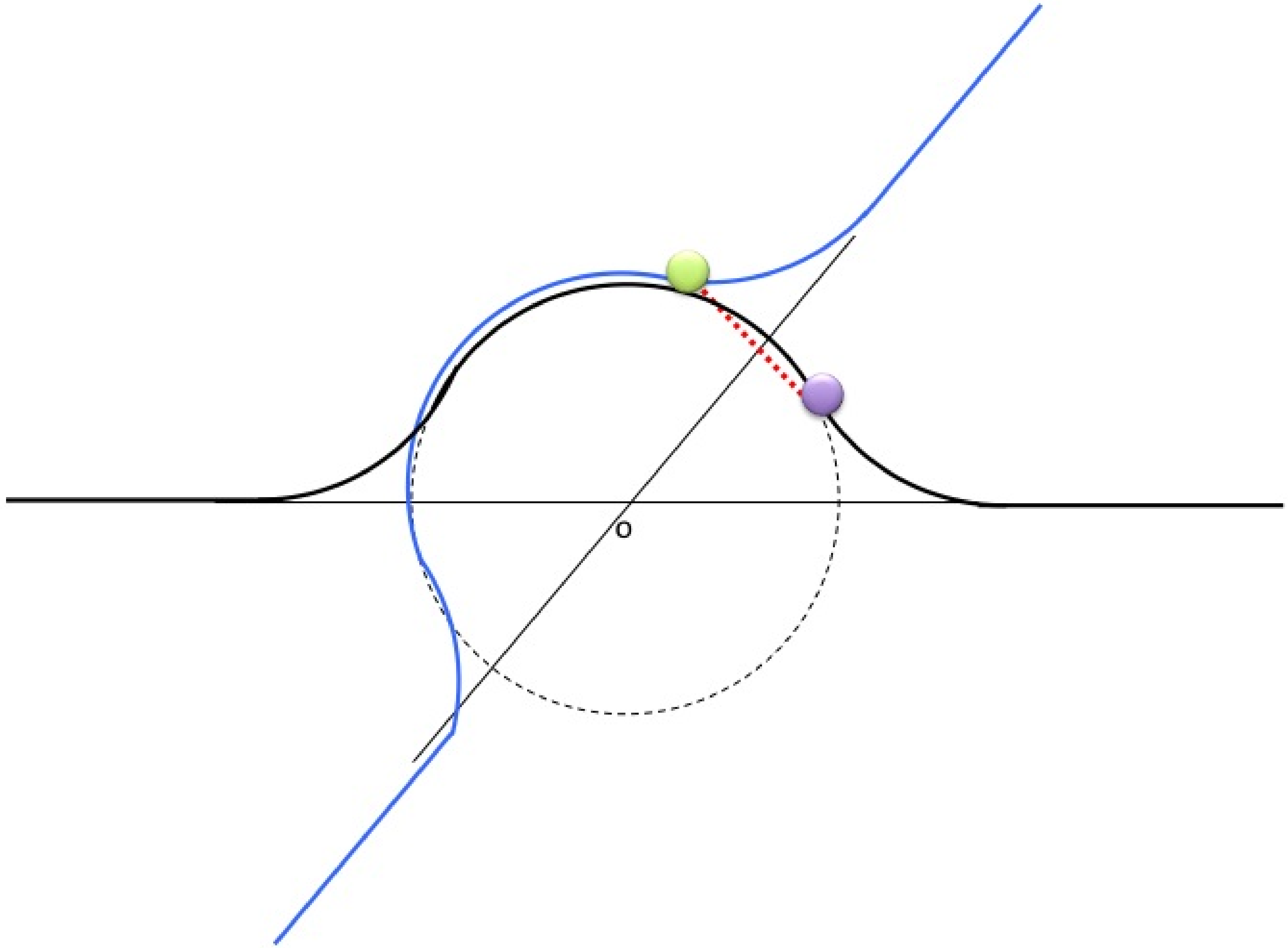
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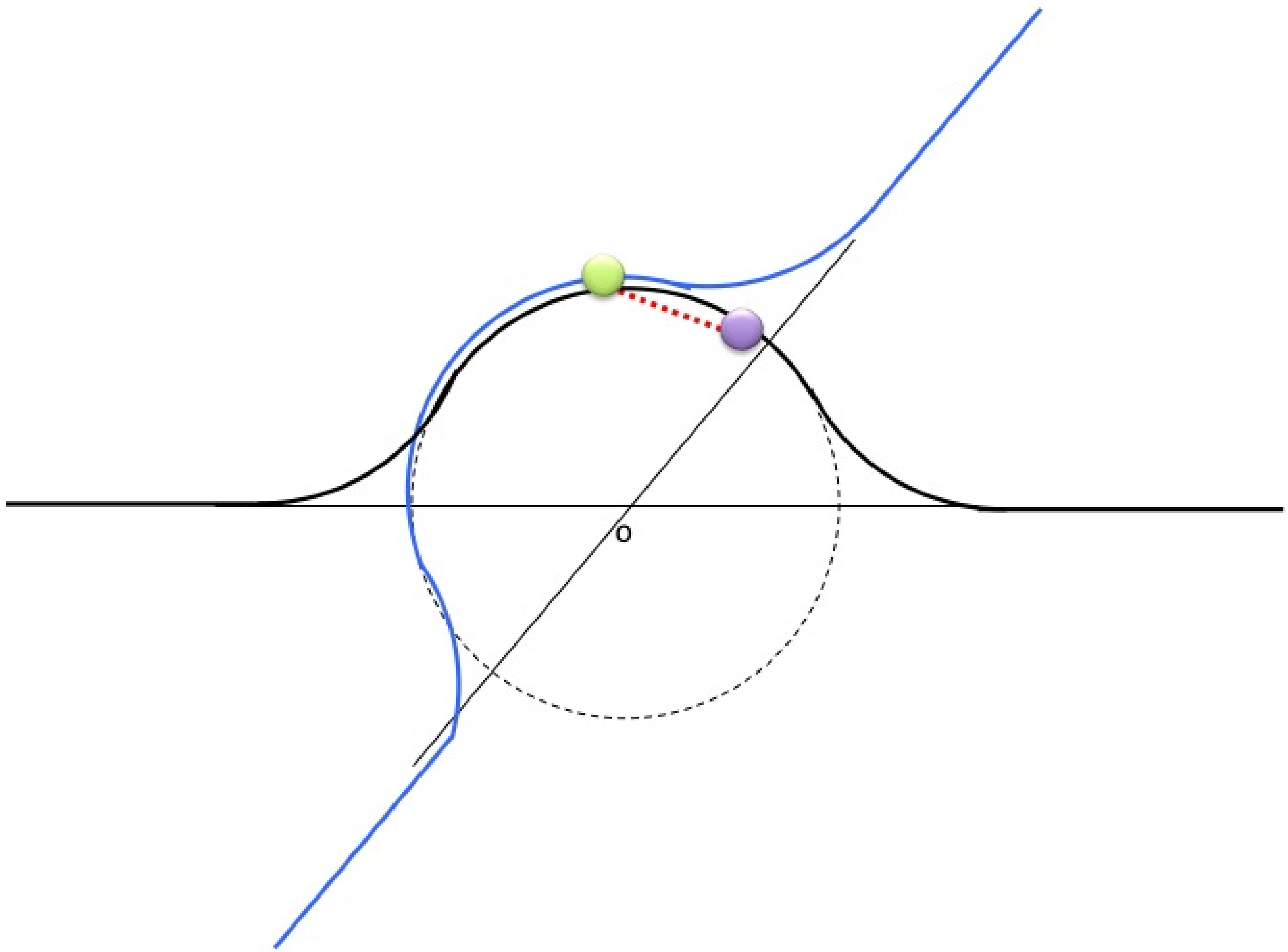
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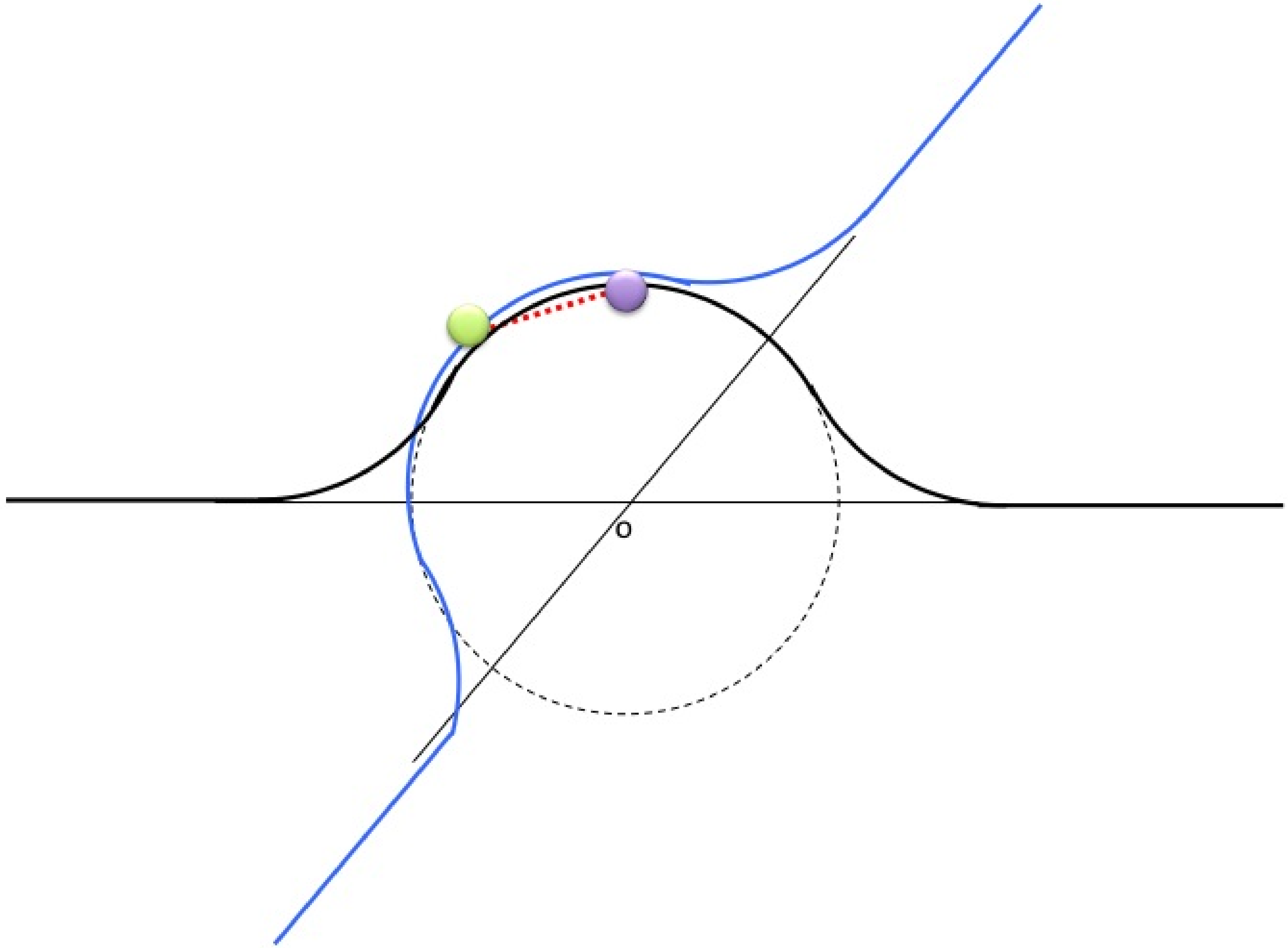
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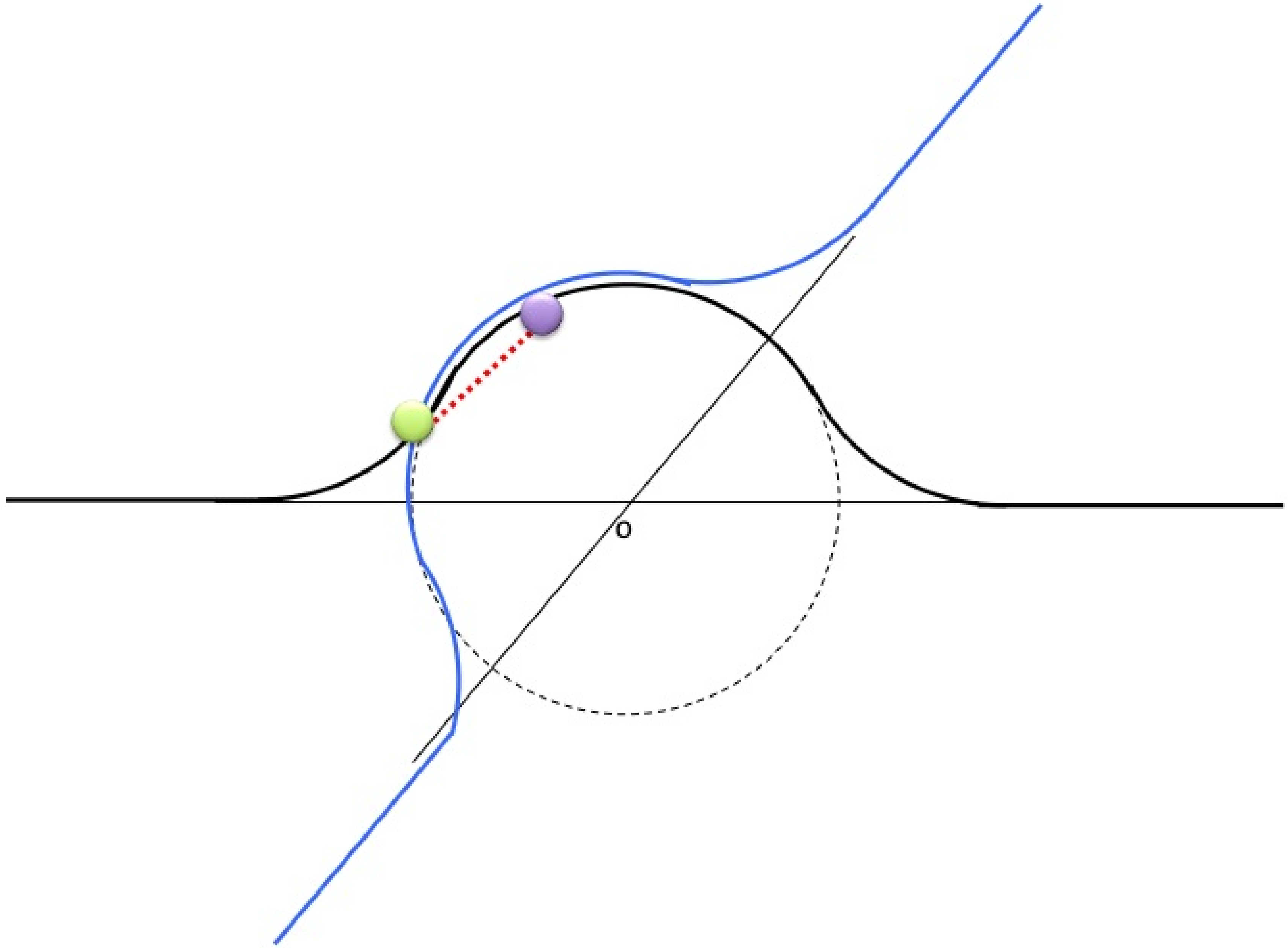
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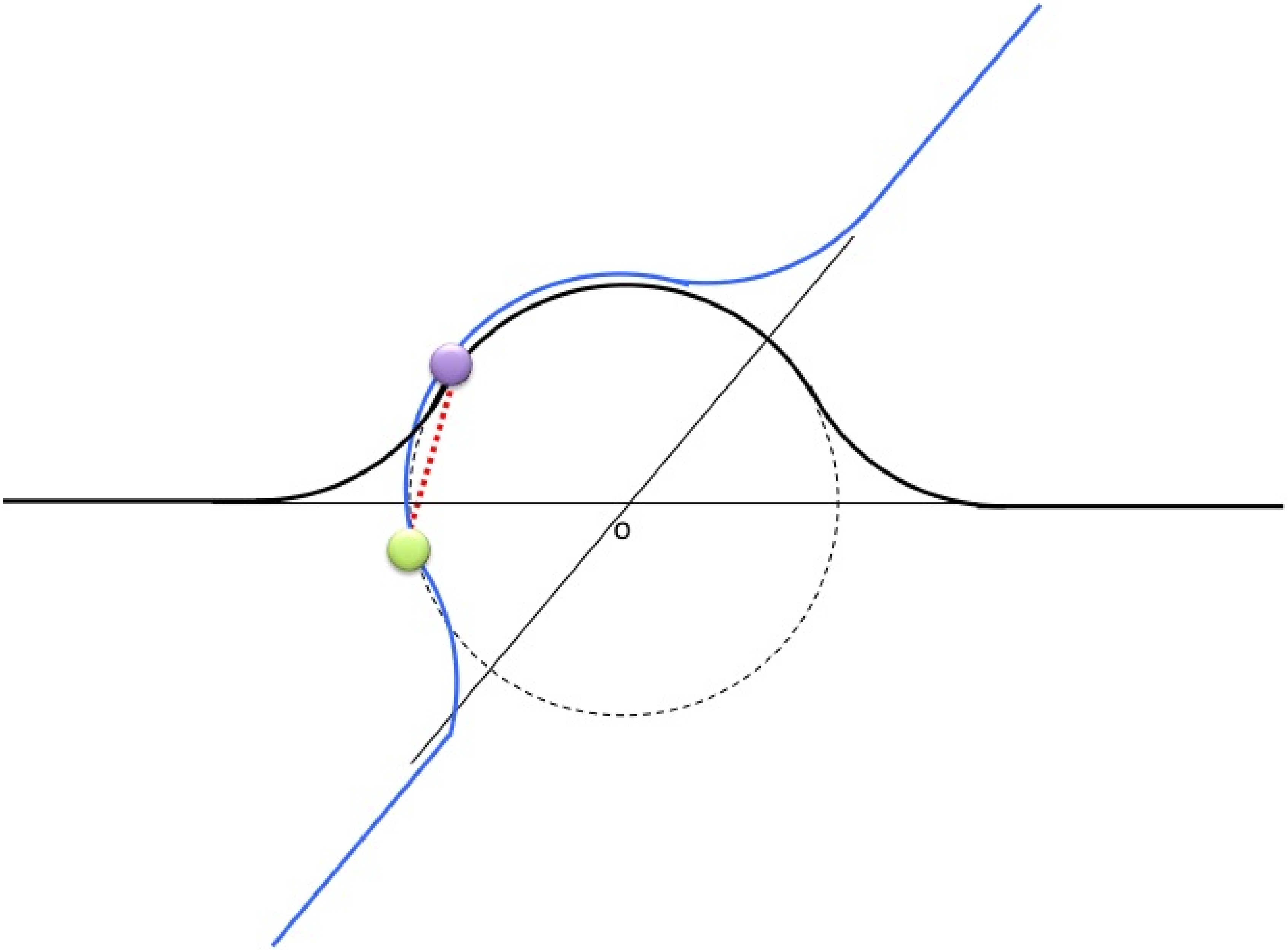
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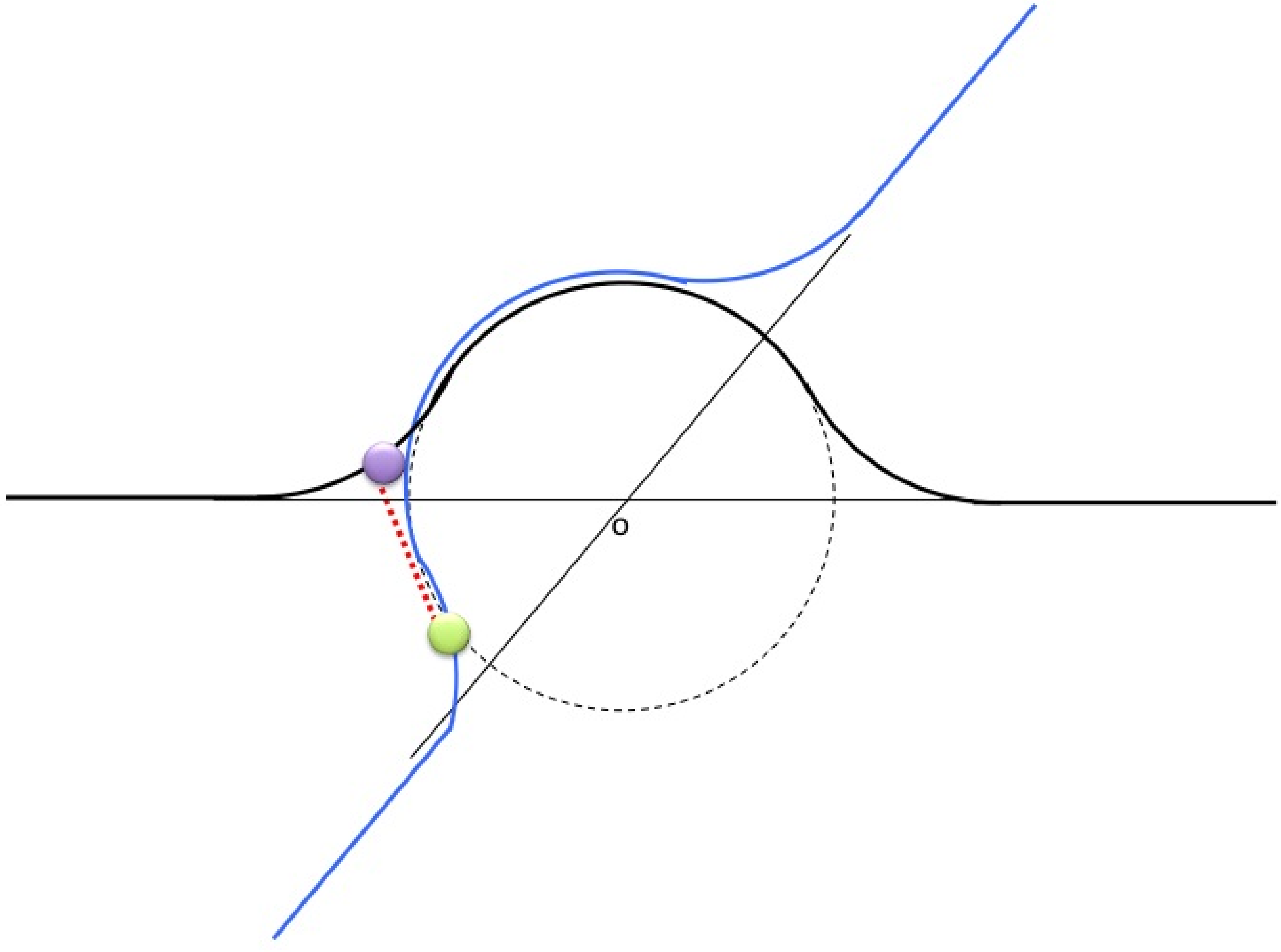
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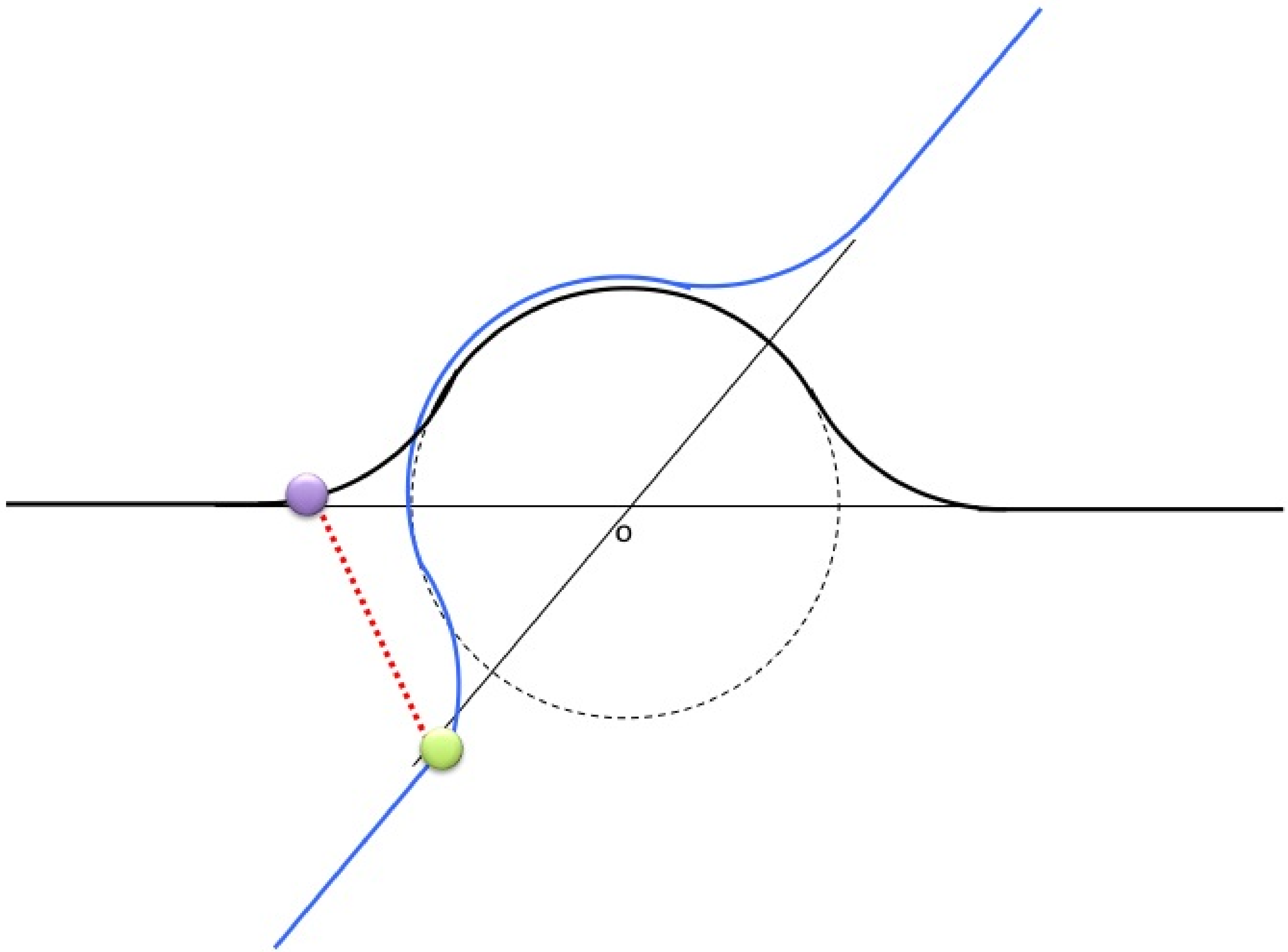
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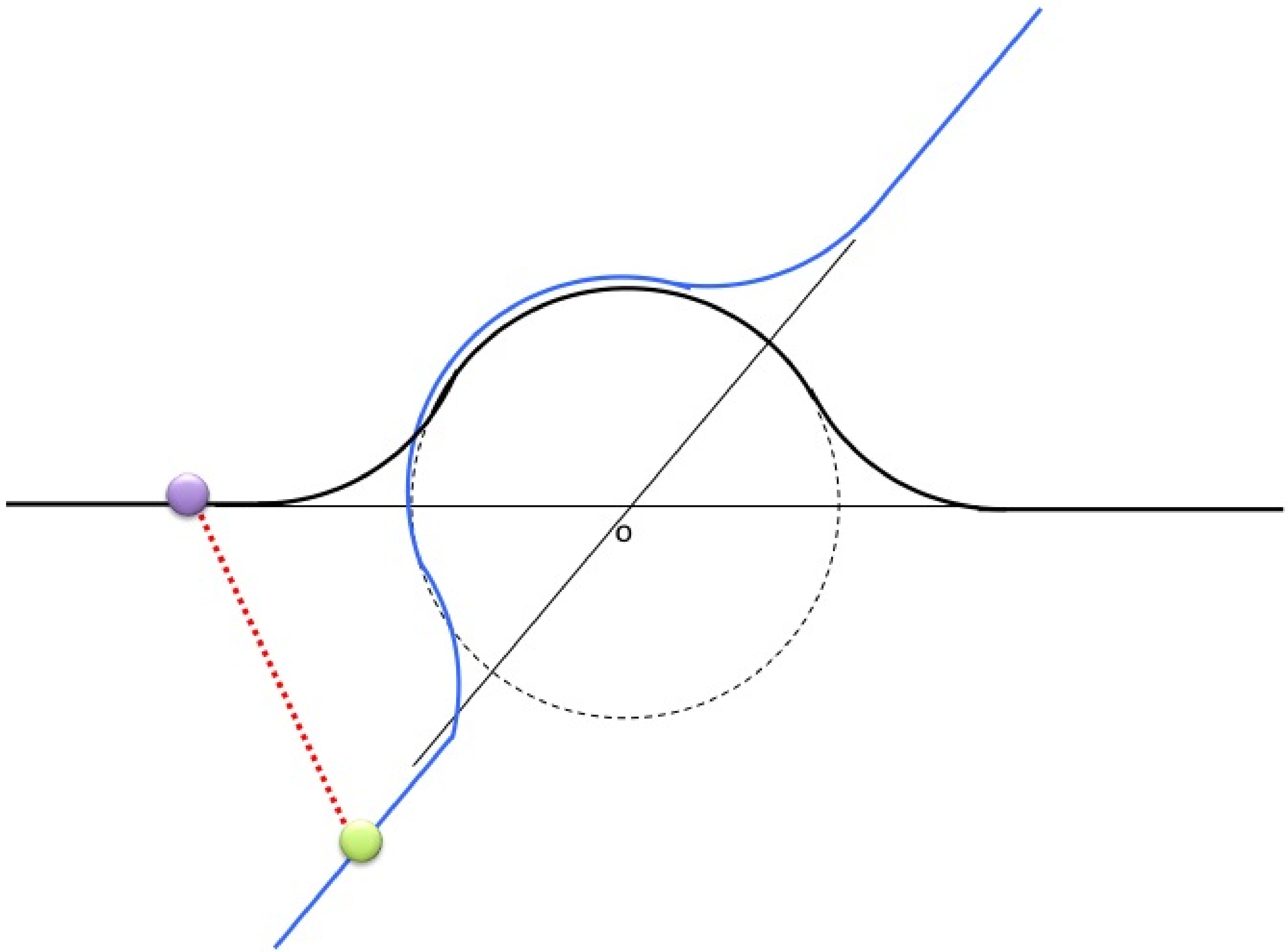
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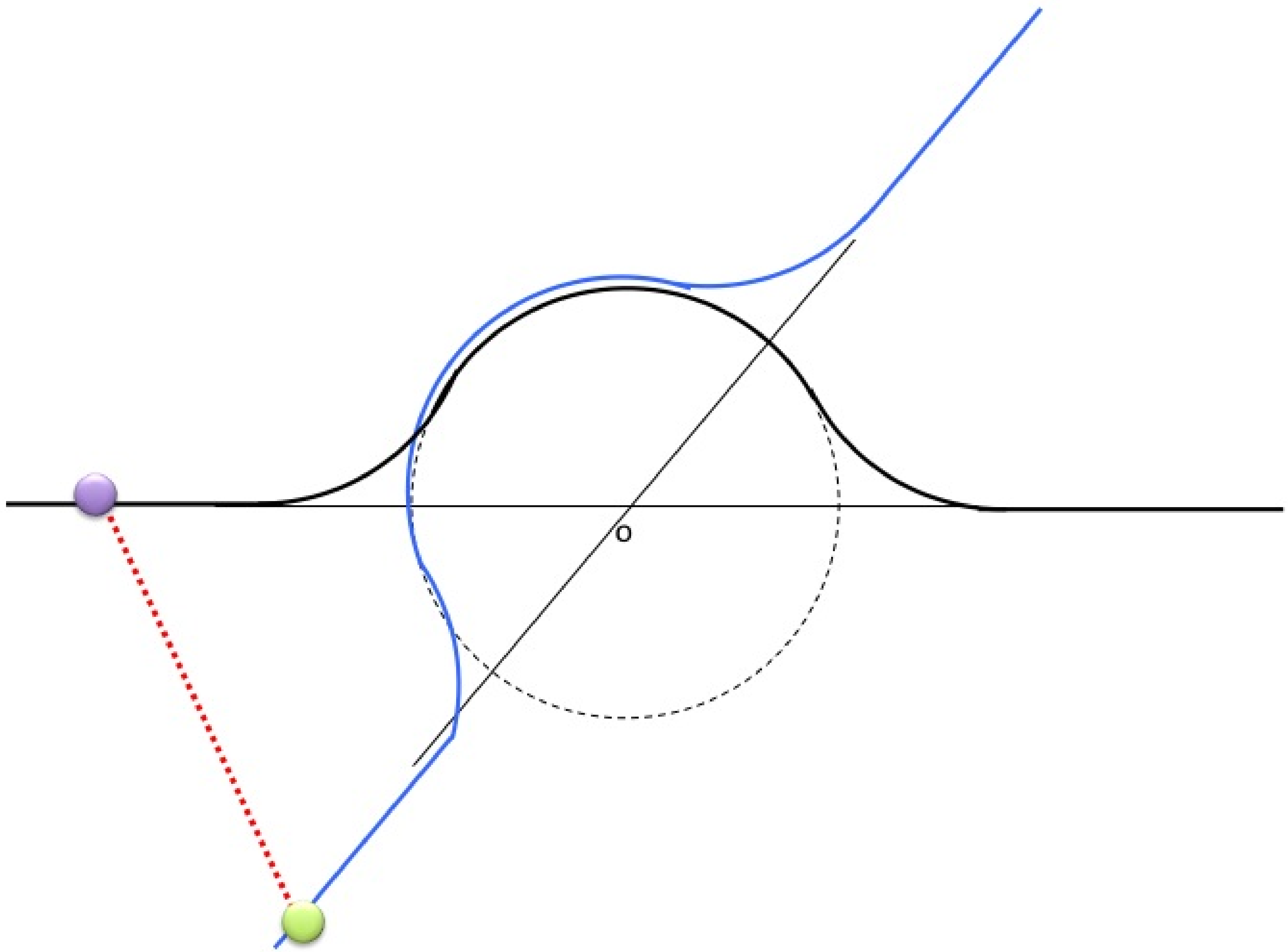
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Making the Maneuver more Precise

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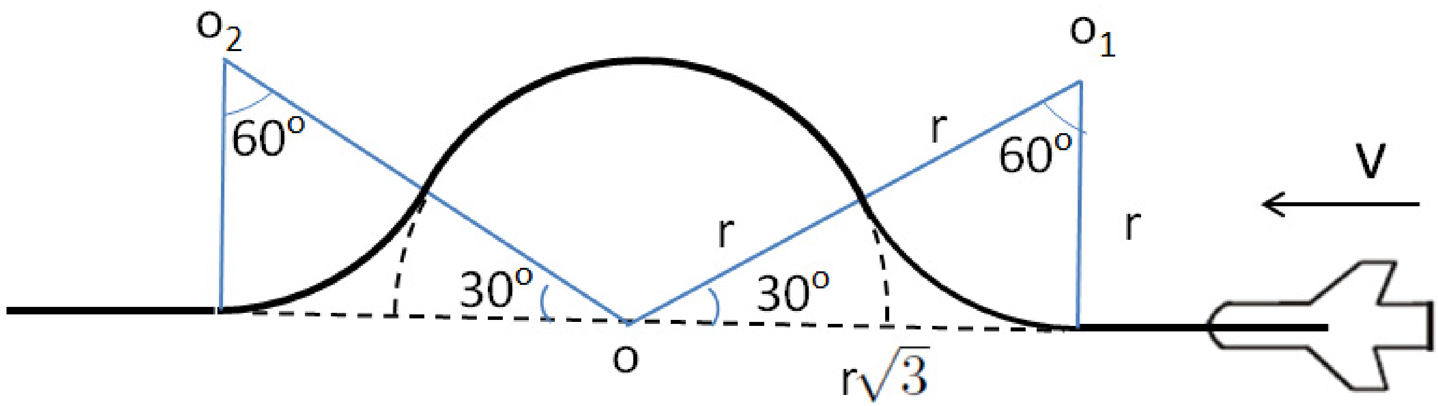

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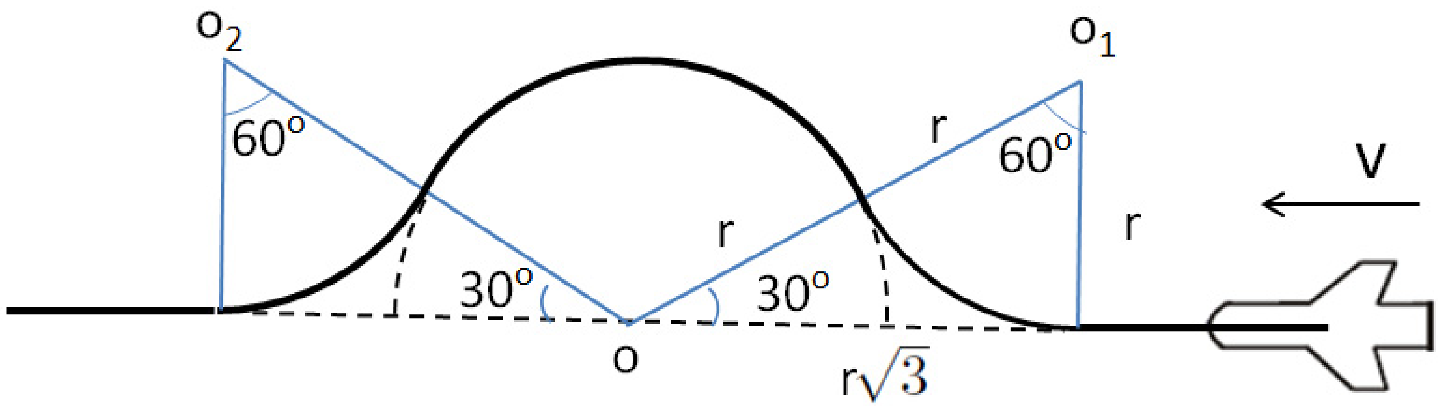

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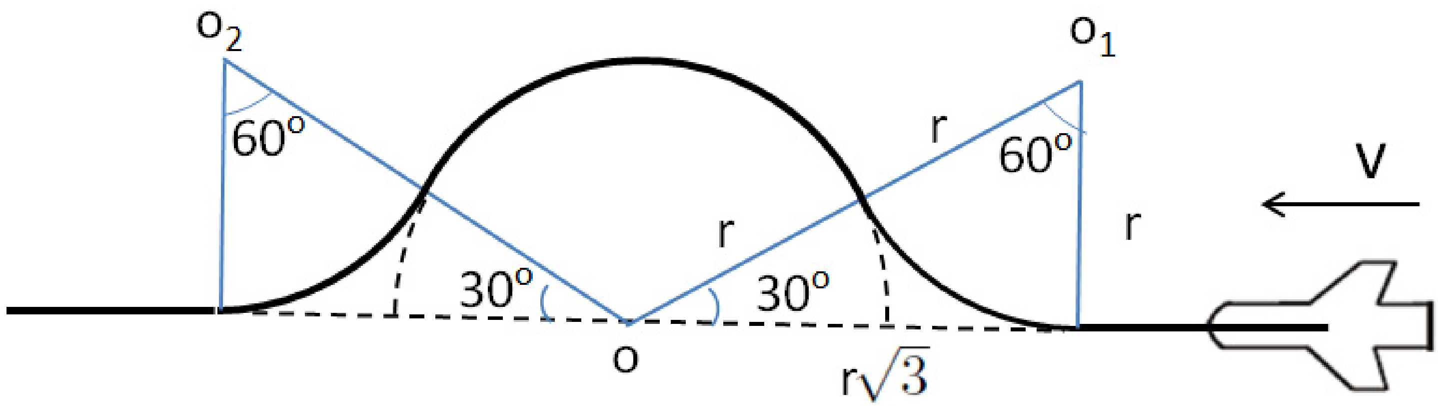

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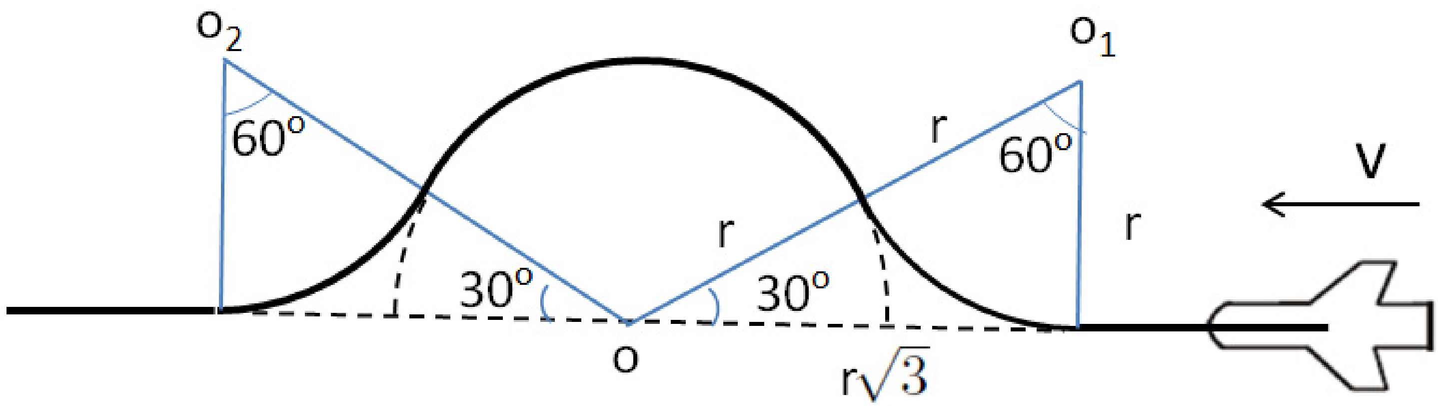

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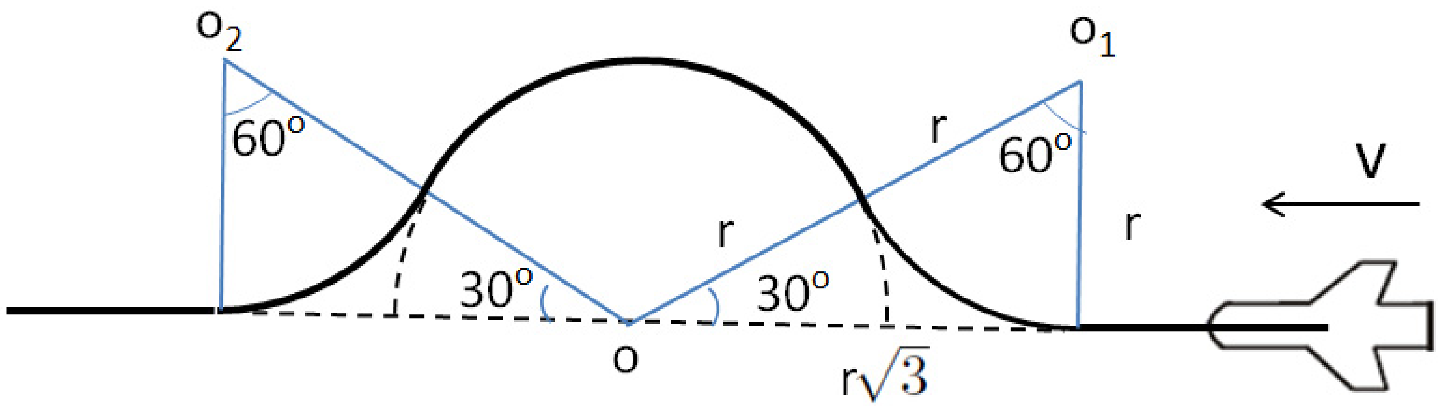

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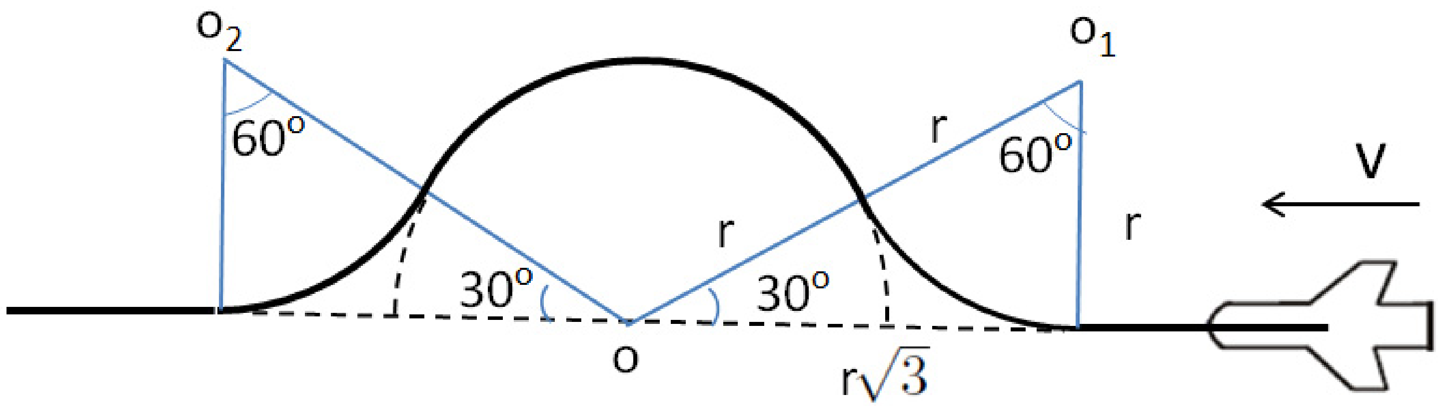

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- We must have then: $\rho_i \geq r\sqrt{3}$ that is $r \leq \frac{\rho_i}{\sqrt{3}}$

- We have then:

$$\frac{p}{2\sin\frac{\phi}{2}} \leq r \leq \frac{\rho_i}{\sqrt{3}}$$

- Here is again the possible interval for the radius r of the circle:

$$\frac{p}{2\sin\frac{\phi}{2}} \leq r \leq \frac{\rho_i}{\sqrt{3}}$$

- We must have then the following for the constants ρ_i, ϕ, and p:

$$2\rho_i \sin\frac{\phi}{2} \geq p\sqrt{3}$$

- ϕ is the angle of the two trajectories

- ρ_i is the initial distance of the two aircrafts to the collision point o

- p is the minimal safety distance between the two aircrafts

axm1: $\phi \in 0 \,..\, \pi$

axm2: $\rho_i \in \mathbb{R}^+$

axm2: $p \in \mathbb{R}^+$

axm4: $2\rho_i \sin\frac{\phi}{2} \geq p\sqrt{3}$

- In this initial model, we are still discrete

- $phase$ corresponds to the various discrete events
- ρ is the common distance of the aircrafts to the collision point o
- r is the circle radius

inv1: $phase \in \{0, 1, 2, 3, 4, 5\}$

inv2: $\rho \in \mathbb{R}^+$

inv3: $r \in \mathbb{R}^+$

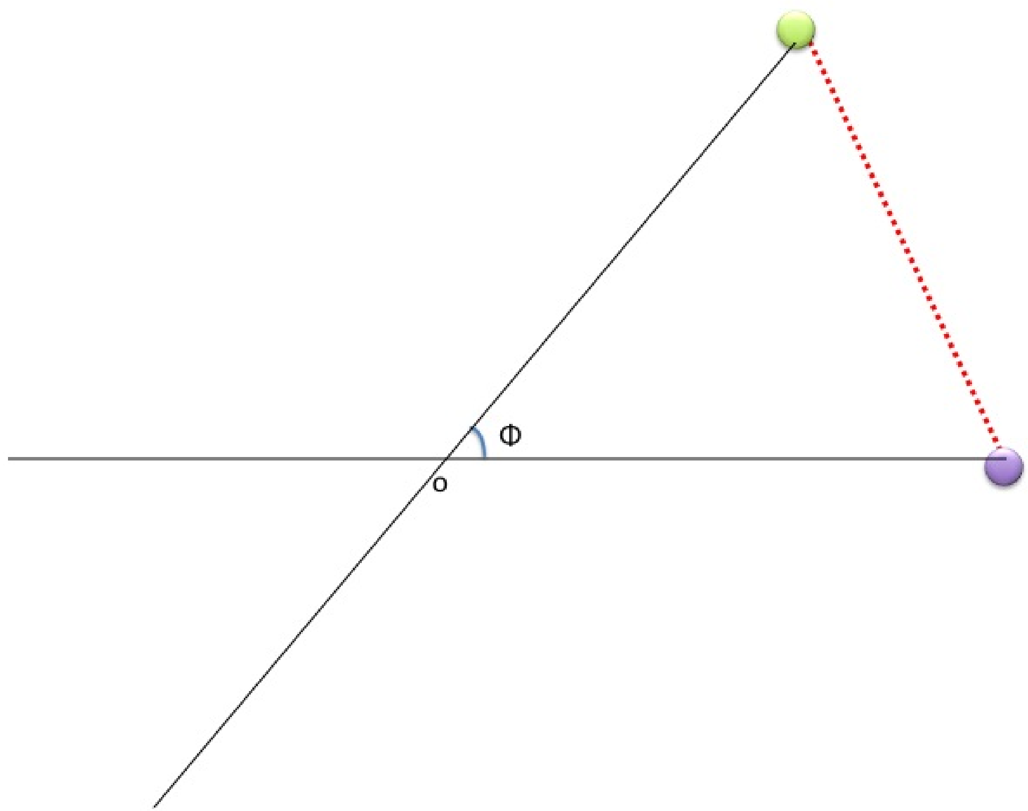
inv4: $2\rho \sin \frac{\phi}{2} \geq p$

- **inv4** is the safety invariant: the minimal authorized distance is p

The Various Discrete Events

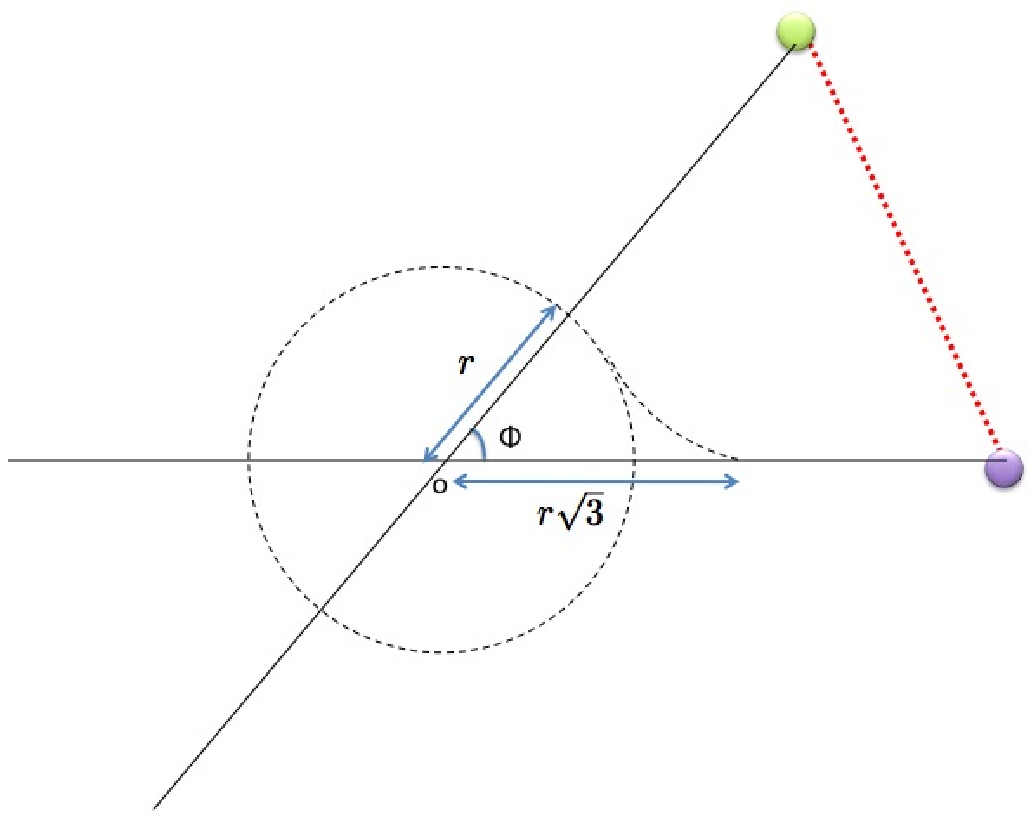
- INIT: initialisation
- agree: choose the radius of the circle
- start: start the maneuver
- enter: entering the circle
- cycle: move on the circle
- leave: leaving the circle

INIT
begin
$\rho := \rho_i$
$phase := 0$
$r :\in \mathbb{R}^+$
end



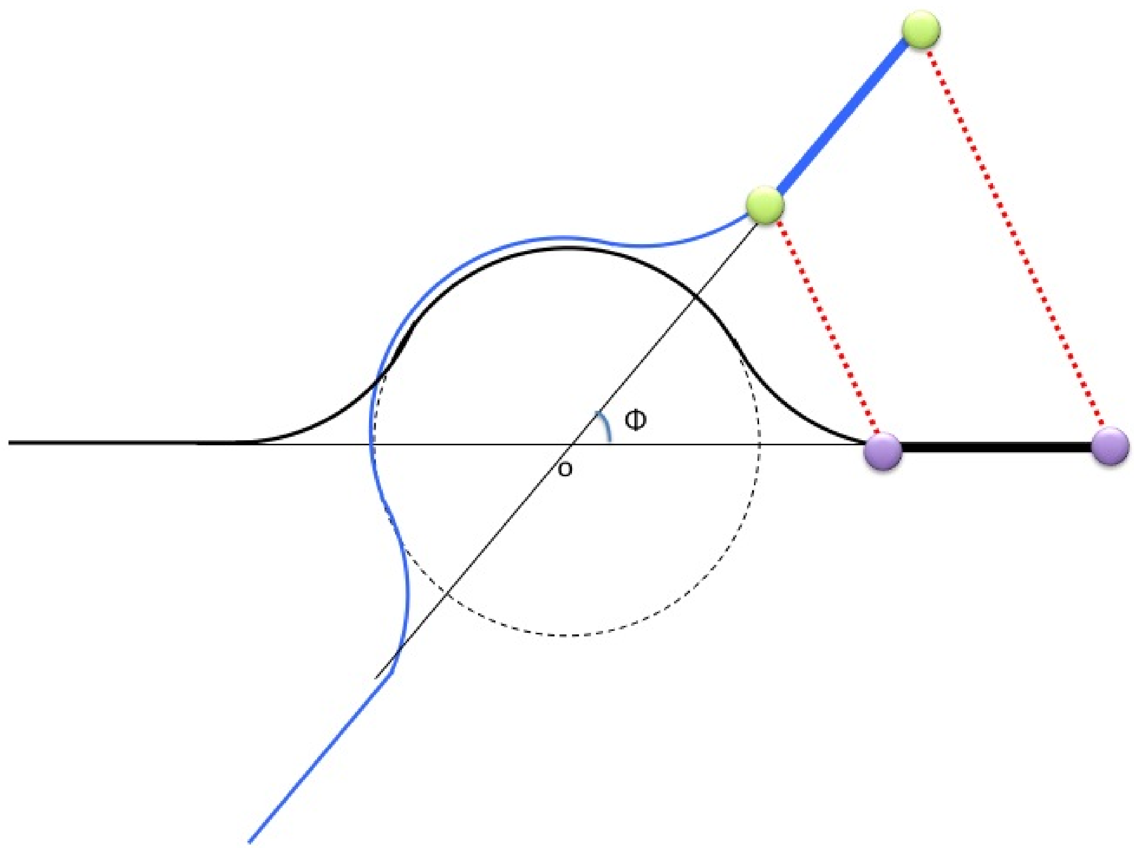
```
agree
  any c where
    phase = 0
    p ≤ 2c sin φ/2
    c√3 ≤ ρ_i
  then
    phase := 1
    r := c
  end
```

Choosing the radius r of the circle



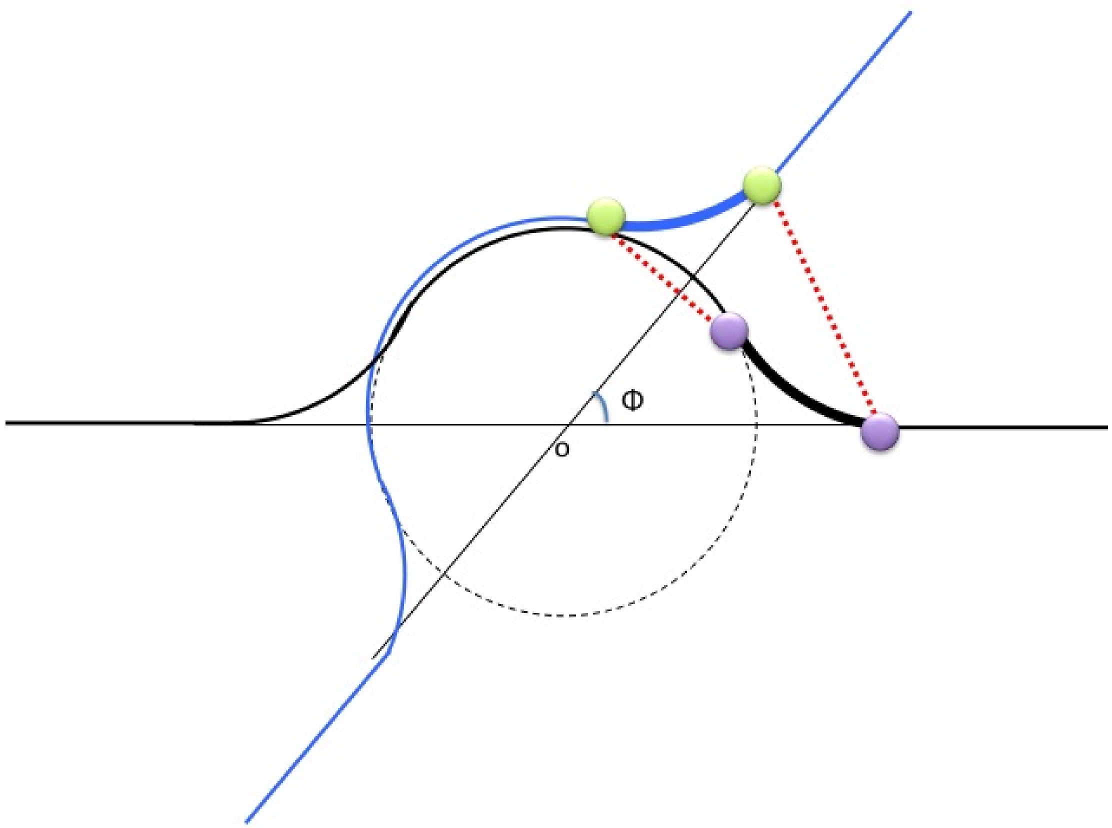
start
when
$phase = 1$
then
$phase := 2$
$\rho := r\sqrt{3}$
end

ρ goes from ρ_i to $r\sqrt{3}$



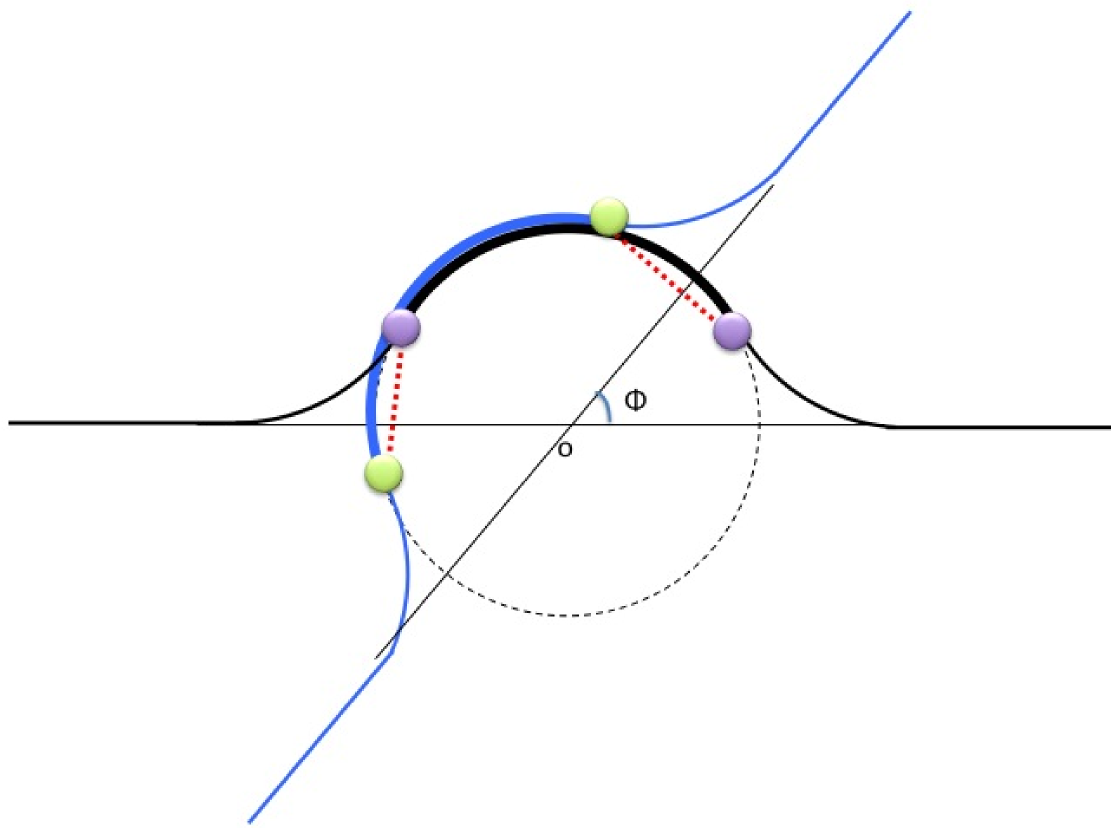
enter
when
$\quad phase = 2$
then
$\quad phase := 3$
$\quad \rho := r$
end

ρ goes from $r\sqrt{3}$ to r



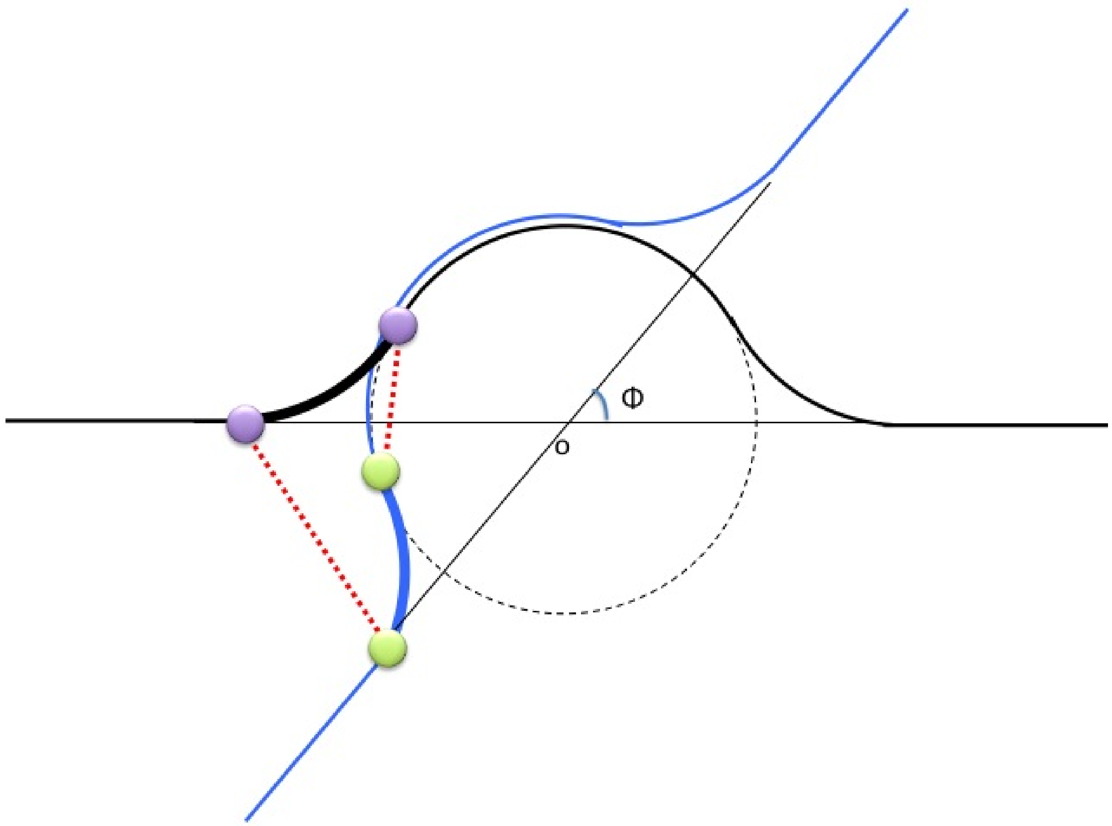
cycle
when
$phase = 3$
then
$phase := 4$
$\rho := r$
end

ρ goes from r to r



leave
when
  $phase = 4$
then
  $phase := 5$
  $\rho := r\sqrt{3}$
end

ρ goes from r to $r\sqrt{3}$



- We introduce the intermediate continuous parts

- We replace ρ by ρ_c (that is $\rho\_continuous$)

- We introduce now, the present time

$$
\begin{array}{ll}
\textbf{inv1\_1:} & \rho\_c \in \mathbb{R}^{+} \rightarrow\!\!\!\!\!\!+ \mathbb{R} \\
\textbf{inv1\_2:} & now \in \mathrm{dom}(\rho\_c) \\
\textbf{inv1\_3:} & \rho = \rho\_c(now) \\
\textbf{inv1\_4:} & \forall t \cdot t \in \mathrm{dom}(\rho\_c) \;\Rightarrow\; 2\rho\_c(t)\sin\frac{\phi}{2} \geq p
\end{array}
$$

- **inv1_3** is the gluing invariant

- **inv1_4** generalises the previous invariant: $2\rho\sin\frac{\phi}{2} \geq p$

(abstract-)INIT

begin

$\rho := \rho_i$

$phase := 0$

$r :\in \mathbb{R}^+$

end

(concrete-)INIT

begin

$\rho\_c := \{0 \mapsto \rho_i\}$

$phase := 0$

$r :\in \mathbb{R}^+$

$now := 0$

end

agree

any c **where**

$phase = 0$

$p \leq 2c \sin \frac{\phi}{2}$

$c\sqrt{3} \leq \rho_i$

then

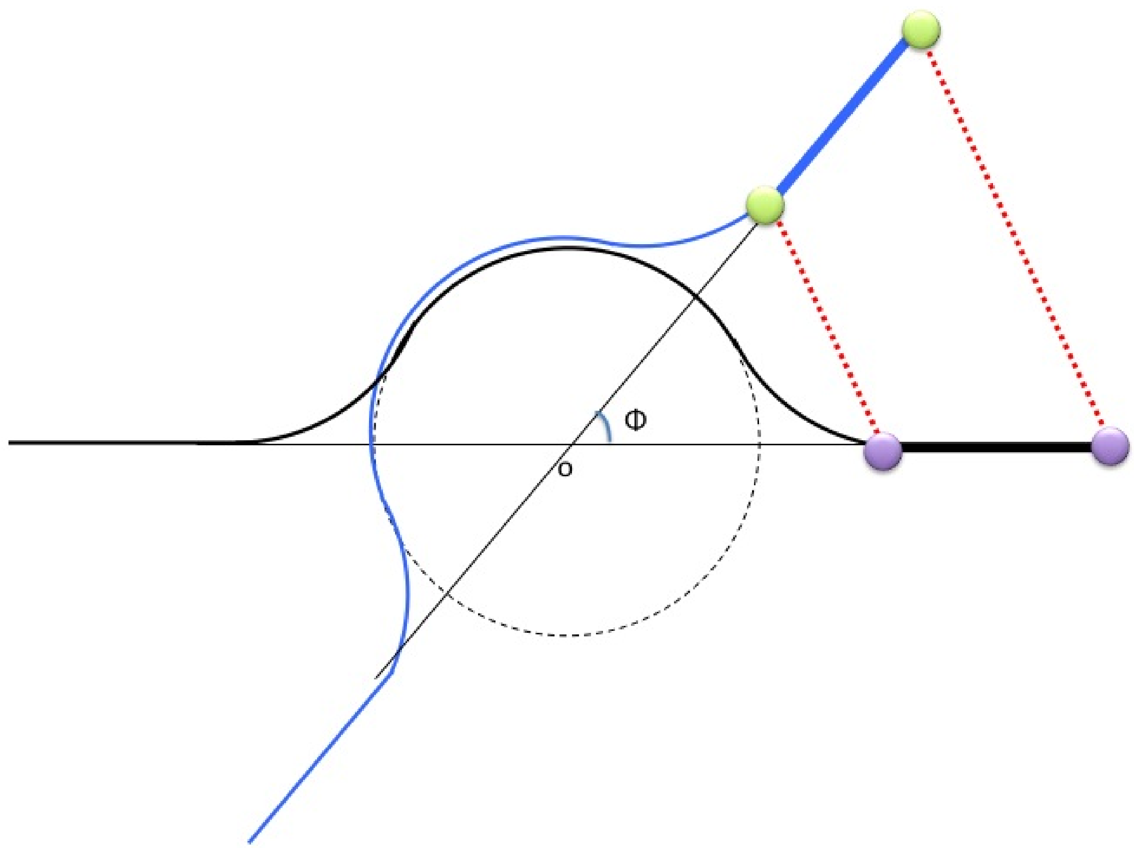
$phase := 1$

$r := c$

end

(abstract-)start
when
$phase = 1$
then
$phase := 2$
$\rho := r\sqrt{3}$
end

(concrete-)start
when
$phase = 1$
then
$phase := 2$
$\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{(\rho_i - r\sqrt{3})}{v} \mid \rho_i - v(t - now)$
$now := now + \frac{(\rho_i - r\sqrt{3})}{v}$
end

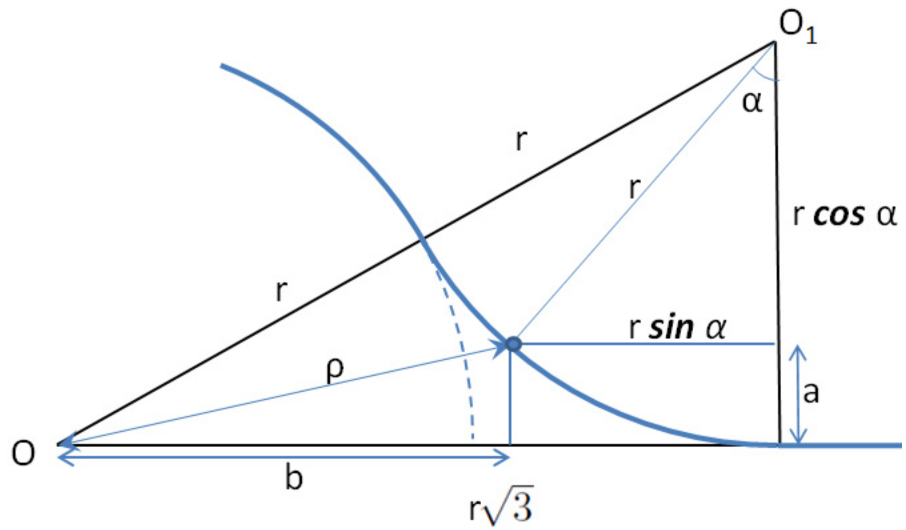


More on the start event

start
  when
    $phase = 1$
  then
    $phase := 2$
    $\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{(\rho_i - r\sqrt{3})}{v} \mid \rho_i - v(t - now)$
    $now := now + \frac{(\rho_i - r\sqrt{3})}{v}$
  end

- $\rho\_c(now) = \rho_i$
- $\rho\_c(now + \frac{(\rho_i - r\sqrt{3})}{v}) = r\sqrt{3}$
- ρ_c decreases linearly from ρ_i to $r\sqrt{3}$
- $\frac{(\rho_i - r\sqrt{3})}{v}$ is the time it takes to fly from ρ_i to $r\sqrt{3}$

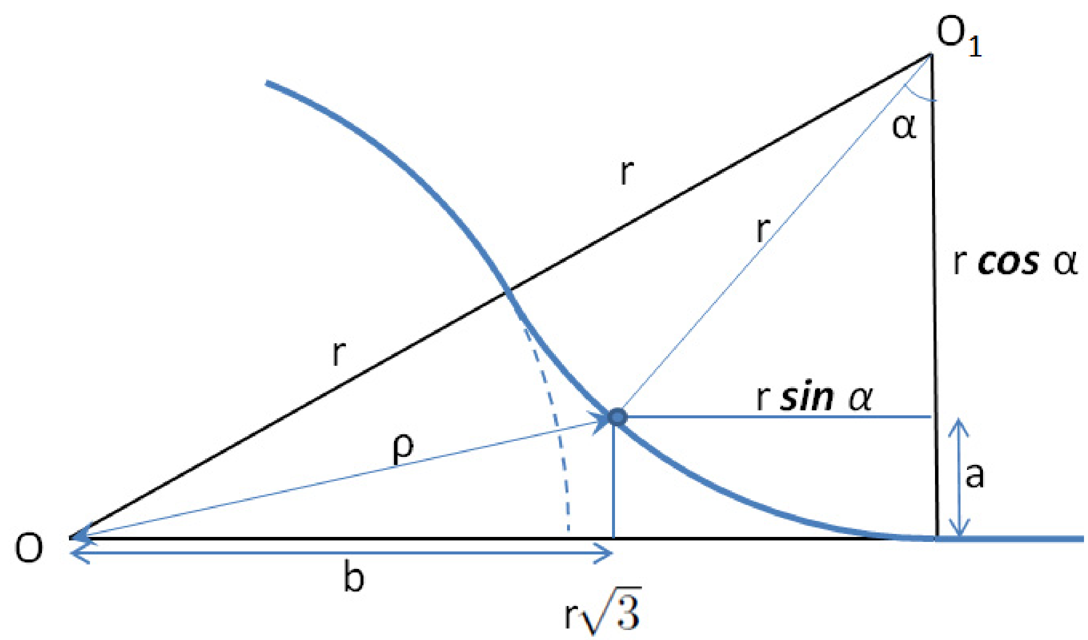
Computing ρ During First Part of Maneuver



$$\rho^2 = a^2 + b^2$$
$$= r^2(1 - \cos\alpha)^2 + r^2(\sqrt{3} - \sin\alpha)^2$$
$$= r^2(5 - 4\cos(\tfrac{\pi}{3} - \alpha))$$
$$\rho = r\sqrt{5 - 4\cos(\tfrac{\pi}{3} - \alpha)}$$

- ρ decreases from $r\sqrt{3}$ to r when α goes from 0 to $\frac{\pi}{3}$.

Computing ρ During First Part of Maneuver



- The angle α increases from 0 to $\frac{\pi}{3}$ during this phase

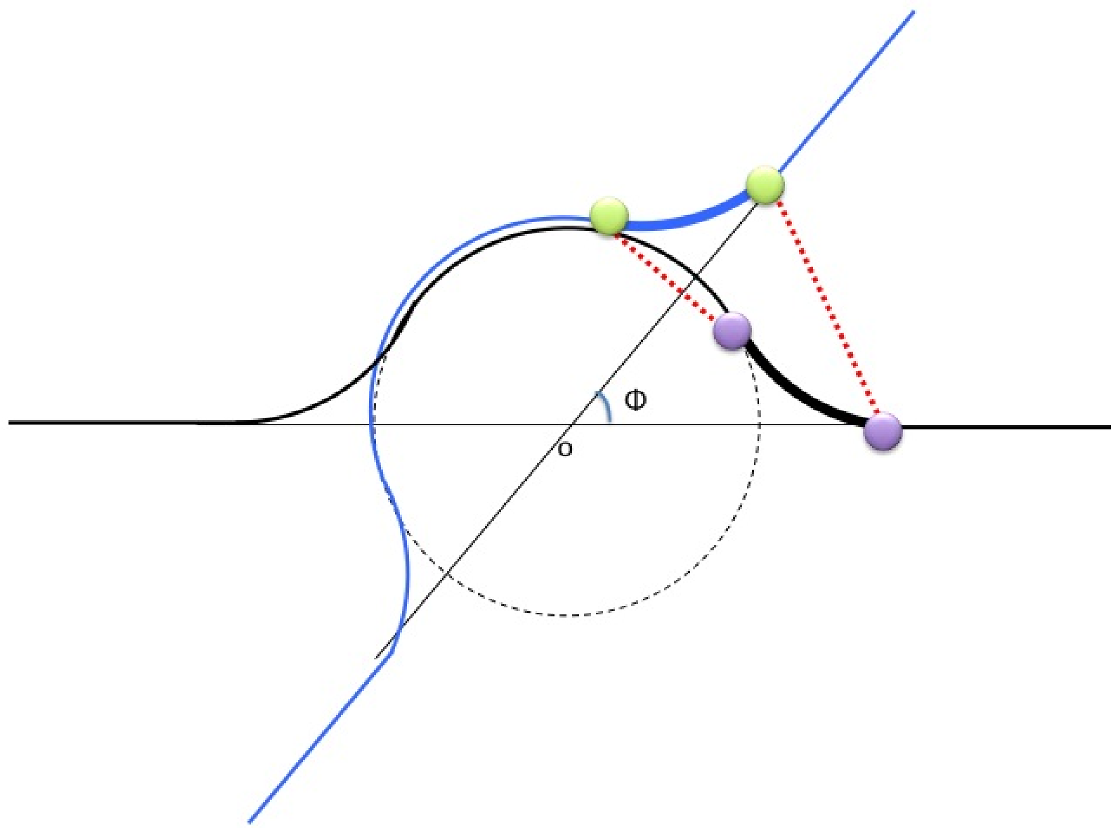
- The distance is $\frac{\pi r}{3}$

- The time to cover this distance is thus $\frac{\pi r}{3v}$

- We have: $\alpha \ = \ \frac{v(t-now)}{r}$

(abstract-)enter
when
$phase = 2$
then
$phase := 3$
$\rho := r$
end

(concrete-)enter
when
$phase = 2$
then
$phase := 3$
$\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{\pi r}{3v} \mid r\sqrt{5 - 4\cos(\frac{\pi}{3} - \frac{v(t-now)}{r})}$
$now := now + \frac{\pi r}{3v}$
end



enter
when
$phase = 2$
then
$phase := 3$
$\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{\pi r}{3v} \mid r\sqrt{5 - 4\cos(\frac{\pi}{3} - \frac{v(t-now)}{r})}$
$now := now + \frac{\pi r}{3v}$
end

- $\rho\_c(now) \;=\; r\sqrt{5 - 4\cos\frac{\pi}{3}} \;=\; r\sqrt{5 - \frac{4}{2}} \;=\; r\sqrt{3}$
- $\rho\_c(now + \frac{\pi r}{3v}) \;=\; r\sqrt{5 - 4\cos(\frac{\pi}{3} - \frac{v\pi r}{r3v})} \;=\; r\sqrt{5 - 4\cos 0} \;=\; r$
- ρ_c decreases non-linearly from $r\sqrt{3}$ to r

Derivation of $\rho\_c(t)$

$$\rho\_c(t) = r\sqrt{5 - 4\cos(\frac{\pi}{3} - \frac{v(t - now)}{r})}$$

Thus

$$\frac{d\rho\_c(t)}{dt} = \frac{4r\sin(\frac{\pi}{3} - \frac{v(t-now)}{r})}{2\sqrt{5 - 4\cos(\frac{\pi}{3} - \frac{v(t-now)}{r})}} \frac{-v}{r}$$

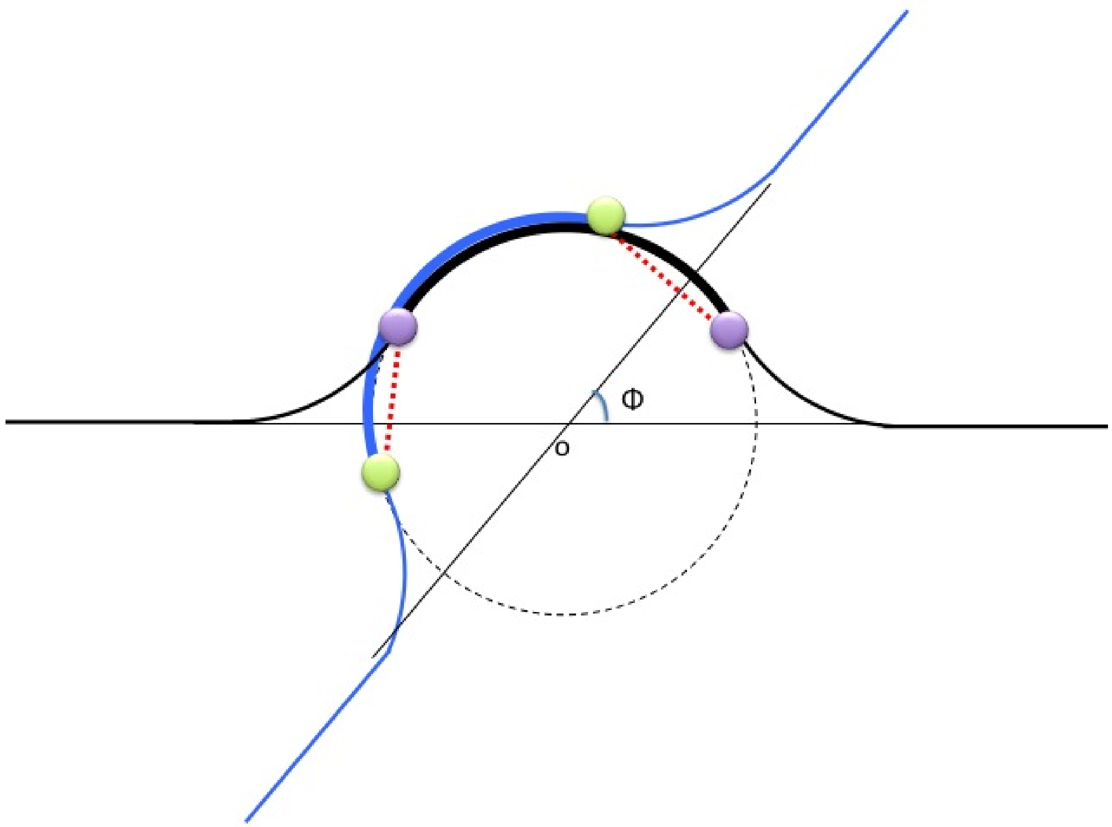
When t increases from now to $now + \frac{\pi r}{3v}$, then the derivative $\frac{d\rho\_c(t)}{dt}$ increases monotonically from $-v$ to 0:

$$\frac{d\rho\_c(t)}{dt}_{t=now} = -v$$

$$\frac{d\rho\_c(t)}{dt}_{t=now+\frac{\pi r}{3v}} = 0$$

(abstract-)cycle
when
$phase = 3$
then
$phase := 4$
$\rho := r$
end

(concrete-)cycle
when
$phase = 3$
then
$phase := 4$
$\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{2\pi r}{3v} \mid r$
$now := now + \frac{2\pi r}{3v}$
end



```
cycle
  when
    phase = 3
  then
    phase := 4
    ρ_c := λt·t ∈ now .. now + 2πr/(3v) | r
    now := now + 2πr/(3v)
  end
```

$$\text{cycle}$$
$$\textbf{when}\quad phase = 3$$
$$\textbf{then}\quad phase := 4,\quad \rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{2\pi r}{3v} \mid r,\quad now := now + \frac{2\pi r}{3v}$$
$$\textbf{end}$$

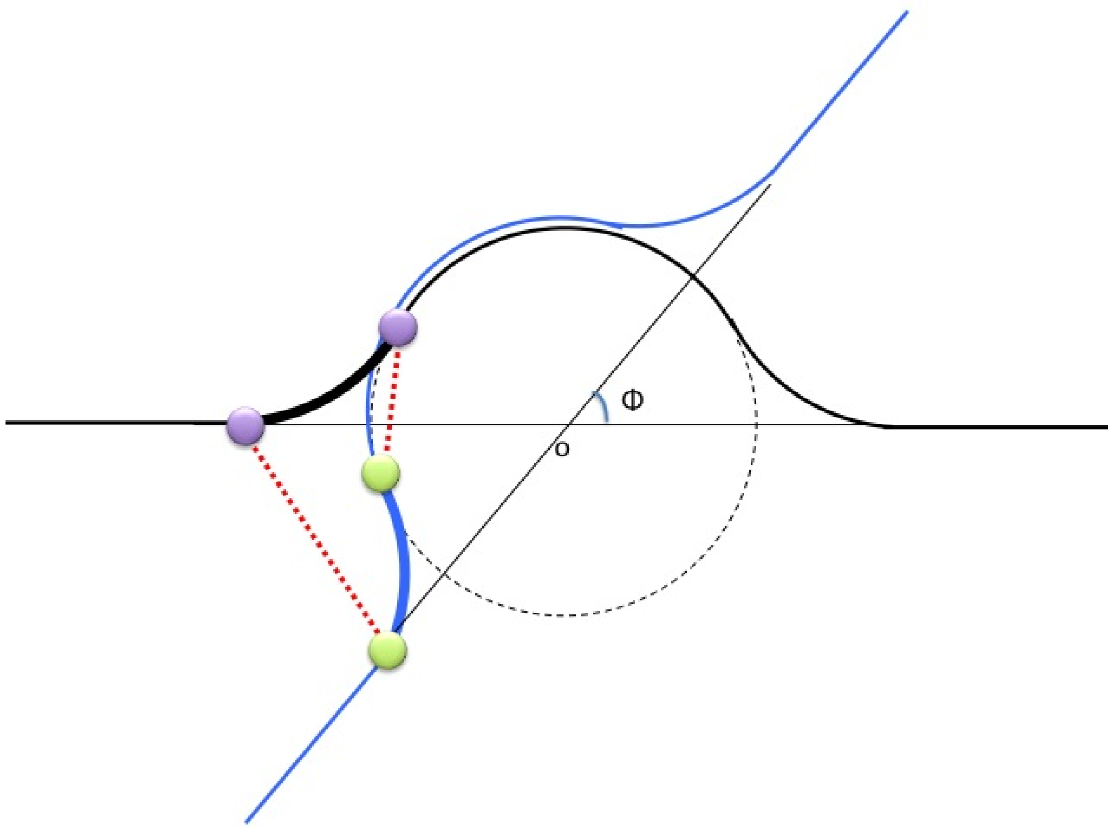
- $\rho\_c(now) \;=\; r$

- $\rho\_c(now + \frac{2\pi r}{3v}) \;=\; r$

- ρ_c remains constant to r

(abstract-)leave
  when
    $phase = 4$
  then
    $phase := 5$
    $\rho := r\sqrt{3}$
  end

(concrete-)leave
  when
    $phase = 4$
  then
    $phase := 5$
    $\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{\pi r}{3v} \mid r\sqrt{5 - 4\cos(\frac{v(t-now)}{r})}$
    $now := now + \frac{\pi r}{3v}$
  end



leave
when
$phase = 4$
then
$phase := 5$
$\rho\_c := \lambda t \cdot t \in now \,..\, now + \frac{\pi r}{3v} \mid r\sqrt{5 - 4\cos(\frac{v(t-now)}{r})}$
$now := now + \frac{\pi r}{3v}$
end

- $\rho\_c(now) \;=\; r\sqrt{5 - 4\cos 0} \;=\; r$
- $\rho\_c(now + \frac{\pi r}{3v}) \;=\; r\sqrt{5 - 4\cos\frac{\pi}{3}} \;=\; r\sqrt{5 - \frac{4}{2}} \;=\; r\sqrt{3}$
- ρ_c increases non-linearly from r to $r\sqrt{3}$

Derivation of $\rho\_c(t)$

$$\rho\_c(t) \;=\; r\sqrt{5 - 4\cos(\frac{v(t-now)}{r})}$$

Thus

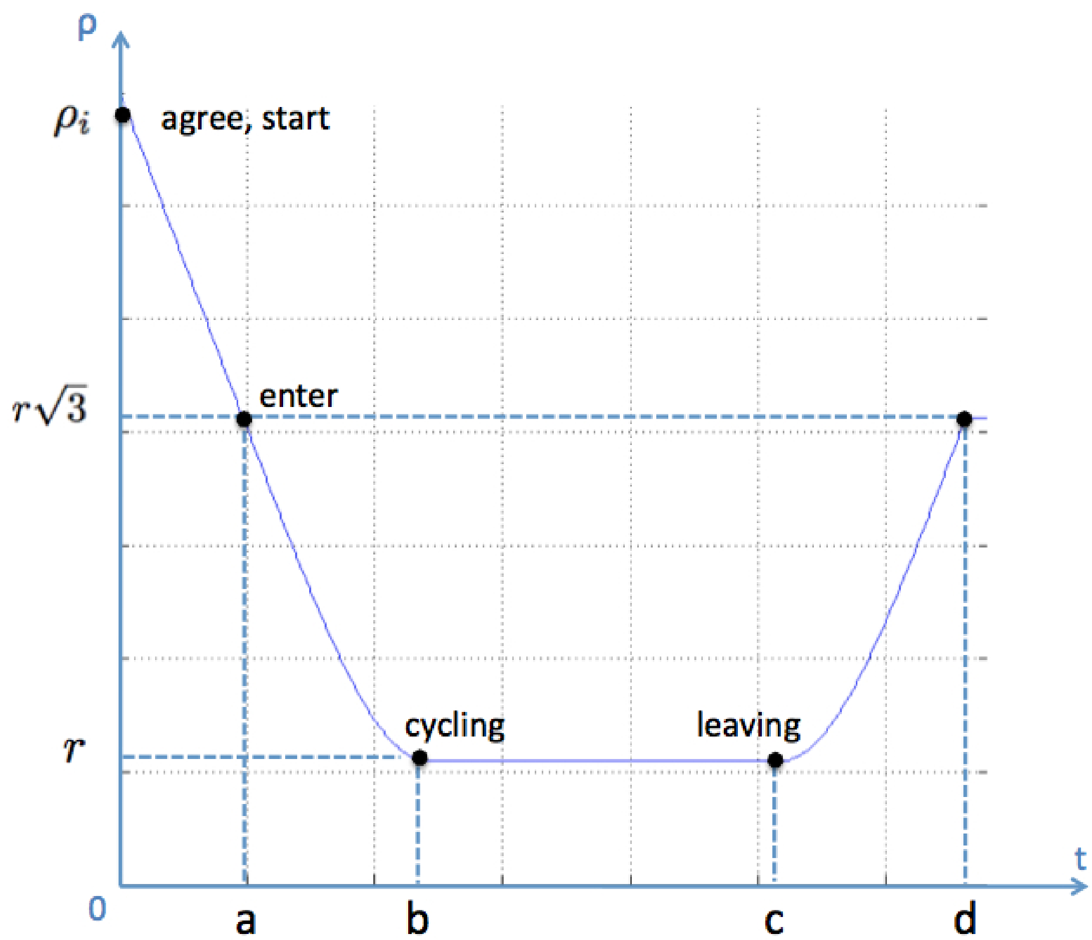
$$\frac{d\rho\_c(t)}{dt} \;=\; \frac{4r\sin(\frac{v(t-now)}{r})}{2\sqrt{5-4\cos(\frac{v(t-now)}{r})}}\frac{v}{r}$$

When t increases from now to $now + \frac{\pi r}{3v}$, then the derivative $\frac{d\rho\_c(t)}{dt}$ increases monotonically from 0 to v:

$$\frac{d\rho\_c(t)}{dt}_{t=now} \;=\; 0$$

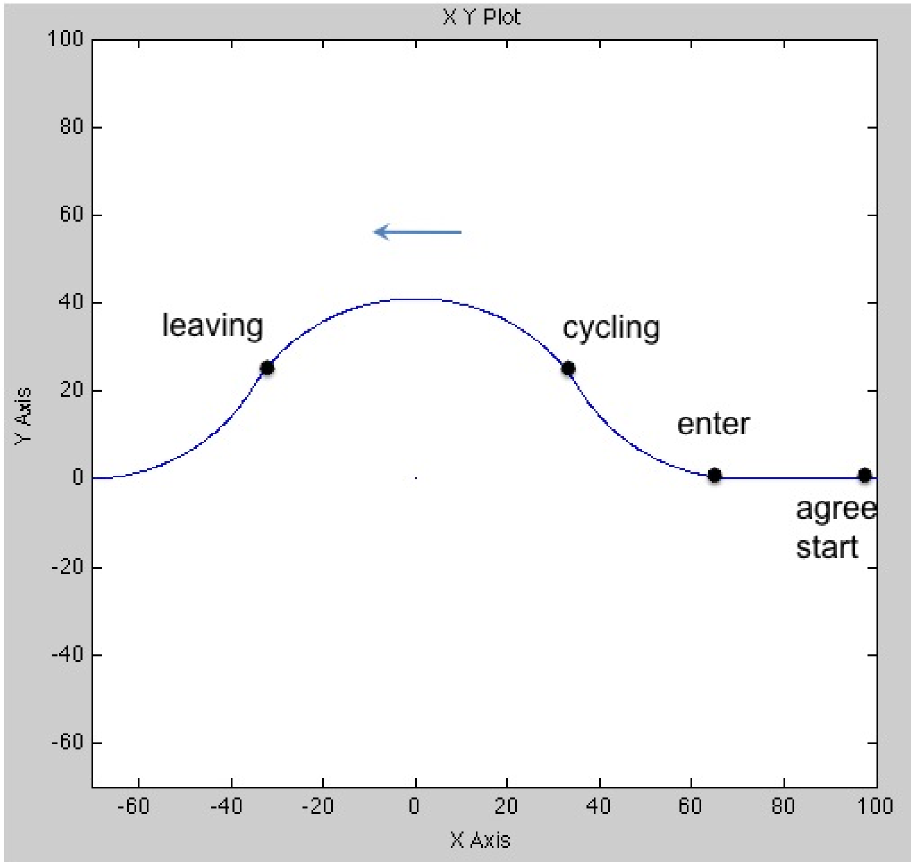
$$\frac{d\rho\_c(t)}{dt}_{t=now+\frac{\pi r}{3v}} \;=\; v$$

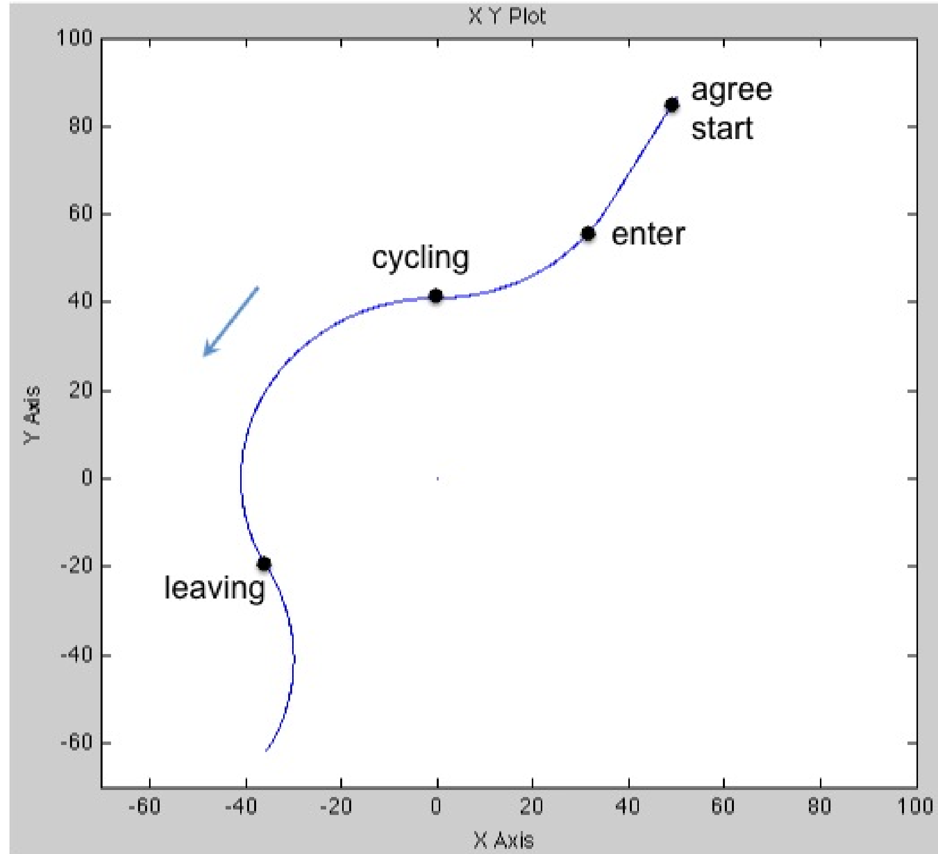
Mathlab/Simulink Output 1: Variation of ρ with Time



$$a = \frac{\rho_i - r\sqrt{3}}{v}, \qquad b = a + \frac{\pi r}{3v}, \qquad c = b + \frac{2\pi r}{3v}, \qquad d = c + \frac{\pi r}{3v}$$

Mathlab/Simulink Output 2: Aircraft Trajectories





Example 2

- Two trains are sent some information by Radio BroadCasting

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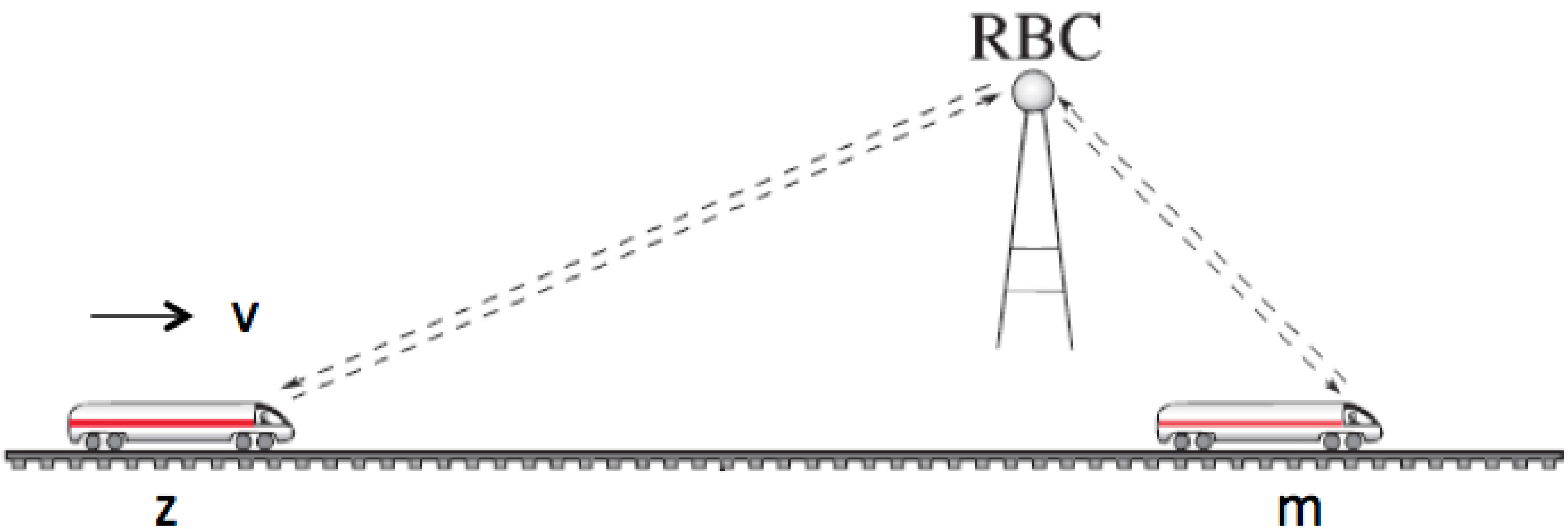

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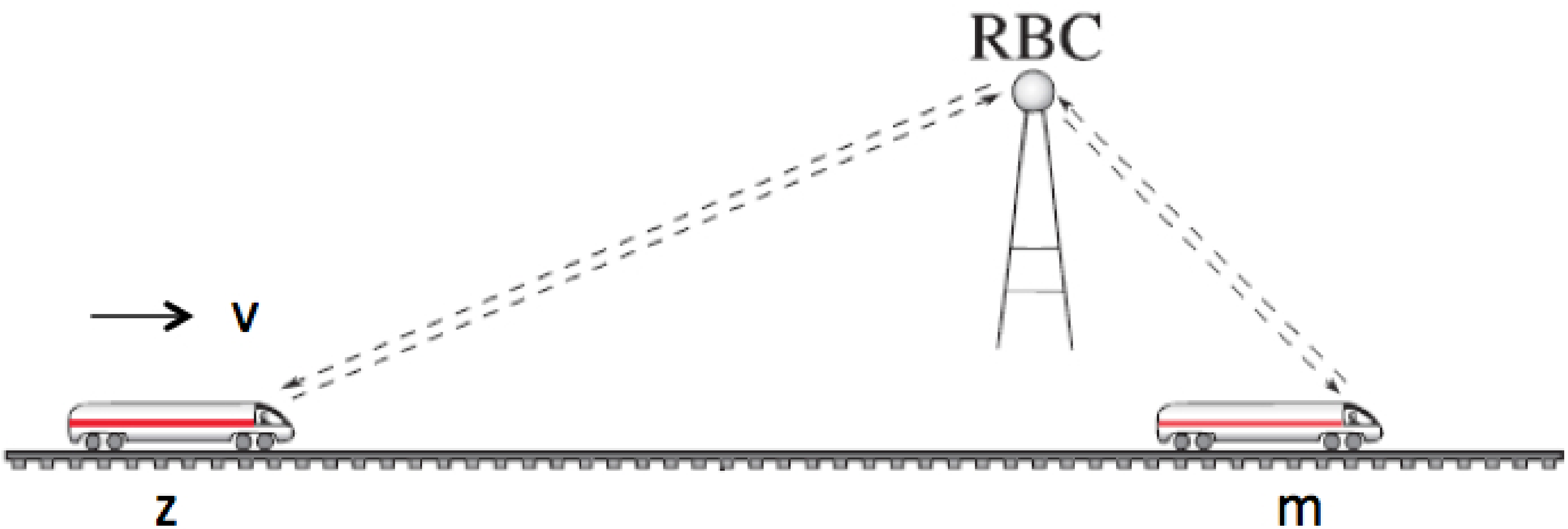

- The second train is in position z

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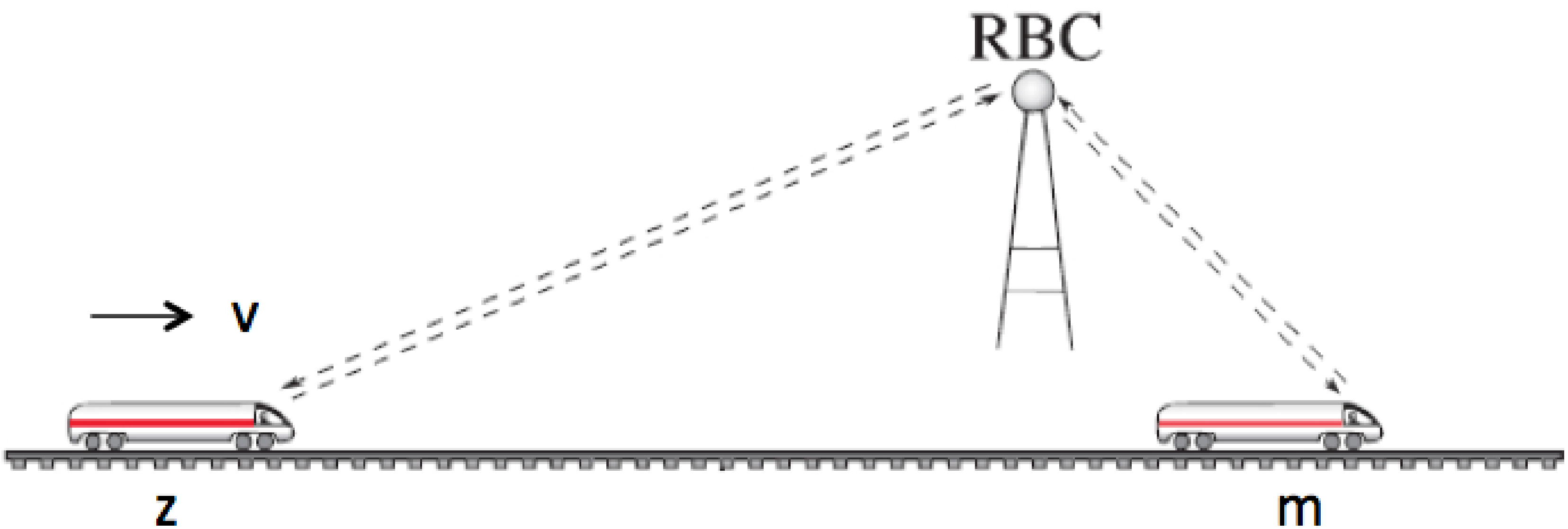

- The second train is in position z

- It is made aware of a position m where it should at the latest stop

Train Control: Constraints and Goal

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- Goal: Calculate the best acceleration at each controller's reaction.

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- This is the main invariant to be maintained

- At each control time (every other ϵ seconds), the invariant to be maintained is:

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$$2b(m - z - v\epsilon - a\frac{\epsilon^2}{2}) \geq (v + a\epsilon)^2$$

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$$2b(m - z - v\epsilon - a\frac{\epsilon^2}{2}) \geq (v + a\epsilon)^2$$

that is

$$2b(m - z) \geq v^2 + (a\epsilon^2 + 2v\epsilon)(a + b)$$

- We must have the following after ϵ seconds:

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- Otherwise, the acceleration should be -b (braking), resulting in:

$$2b(m - z) \geq v^2$$

- After the choice of acceleration, A or -b, the speed of the train is:

$$v + a\epsilon$$

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- We have thus three different controller decisions:
    - decision 1: acceleration -b
    - decision 2: acceleration A
    - decision 3: acceleration 0

- If the speed and position are v and z, then after ϵ seconds:

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- the new speed of the train will be:

    - drive 1: if $v + a\epsilon \geq 0$ then $v + a\epsilon$

    - drive 2: if $v + a\epsilon < 0$ then 0

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- the new position of the train will be:

  - drive 1: if $v + a\epsilon \geq 0$ then $z + v\epsilon + a\frac{\epsilon^2}{2}$

  - drive 2: if $v + a\epsilon < 0$ then $z + \frac{v^2}{2b}$ (the train stops after time $\frac{v}{b}$)

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Conclusion and Future Work

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- Continuous variables are not defined by differential equations

Thank you for listening